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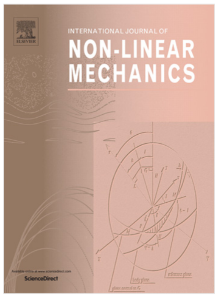


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# Bifurcation study of a chaotic model variable-length pendulum on a vibrating base

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**Abstract**

It is known that in the dissipative system of an inverted pendulum of constant length on an oscillating base, a cascade of bifurcations arises, leading to chaos. In this paper, the appearance of chaotic behavior of a conservative variable-length pendulum on a vibrating base near the upper equilibrium position at high vibration frequencies and small amplitudes of harmonic oscillations of the length of the pendulum and the point of its suspension is discovered and investigated. As a mathematical model, a non-autonomous averaged second-order system with dissipation near resonance 1:2 between the oscillation frequencies of the length and oscillations of the suspension point is used. A numerically-analytical bifurcation study of an autonomous control system and a non-autonomous dissipative system is performed at a decrease in the dissipation coefficient to zero. Cascades of bifurcations of limit cycles in the neighborhood of the upper equilibrium position, leading to the formation of a chaotic attractor, are found. The presence of dynamic chaos is proved by graphs and maps of the largest Lyapunov exponent, by maps of dynamic regimes and bifurcation diagrams.

*Keywords:* variable-length pendulum, resonance, Lyapunov exponent, bifurcation diagram, dynamic chaos
MSC[2010] 34C15, 70K65, 37H20, 74H65

## 1. Introduction

A pendulum with an oscillating suspension [1] is a model for investigating both the regular and the chaotic behavior of nonlinear dynamical systems. Studies of the chaotic behavior of a parametrically excited pendulum are carried out for more than thirty years by approximate analytical, numerical, and experimental methods in various formulations [2 - 8]. The transition to chaos is studied by the methods of bifurcation theory, the study of attraction basins [3], [4, 6], the construction of maps of the Poincare map, and the calculation of the largest Lyapunov exponent [5, 7, 8].

A variable-length pendulum was considered in [9 -18]. Complex oscillations of a variable-length pendulum with dissipation take place in crane models [10]. For a periodic change in length as a function of the oscillation phase, the chaotic dynamics of swings or a pendulum of variable length on a fixed base has been studied [11,12]. It is shown

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