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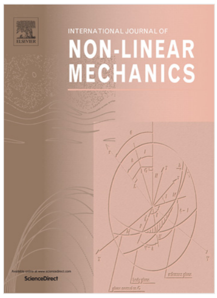


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# BOUNDED AND UNBOUNDED SOLUTIONS OF A DISCONTINUOUS OSCILLATOR AT RESONANCE

HEBAI CHEN[1,2], JINQIAO DUAN[3]

ABSTRACT. First, we have studied a specific discontinuous differential equation for a smooth and discontinuous (SD) oscillator $\ddot{x} + x\ (1 - 1/\sqrt{a^2 + x^2}) = p(t)$, where $p(t)$ is a given $2\pi$-periodic forcing function and $a$ is a real parameter. When the forcing $p(t) \in C^7(\mathbb{R}/2\pi\mathbb{Z})$ and $\left|\int_0^{2\pi} p(t)e^{-it}dt\right| < 4$, all solutions of the oscillator are shown to be bounded, i.e.,

$$\lim_{|t|\to+\infty} \left\{x(t)^2 + \dot{x}(t)^2\right\} < +\infty.$$

Moreover, the oscillator has at least one harmonic solution. When the forcing $p(t) \in C^0(\mathbb{R}/2\pi\mathbb{Z})$ and $\left|\int_0^{2\pi} p(t)e^{-it}dt\right| \geq 4$, all solutions of the oscillator are unbounded, i.e.,

$$\lim_{|t|\to+\infty} \left\{x(t)^2 + \dot{x}(t)^2\right\} = +\infty.$$

The two conditions are sharp because they complement each other. Moreover, a bifurcation that connects the bounded solutions with unbounded ones occurs, and 4 is a bifurcation value. Inspired by this special discontinuous oscillator, we have found the conditions for the existence of bounded and unbounded solutions for a more general discontinuous oscillator. Finally, we present interesting physical models to illustrate our results.

## 1. Introduction

Littlewood in [18, 19, 20] suggested to study the bounded and unbounded solutions of the following Liénard system for an oscillator $\ddot{x} + g(x) = p(t)$, with a nonlinear interaction $g$ and periodic forcing $p$. A solution of this type of differential equation is called "bounded" if

$$\lim_{|t|\to+\infty} \left\{x(t)^2 + \dot{x}(t)^2\right\} < +\infty.$$

For example, the periodic solutions and quasi-periodic solutions are bounded solutions, where the definition of quasi-periodic solutions is shown in Section 2. A solution is called "unbounded" if

$$\lim_{|t|\to+\infty} \left\{x(t)^2 + \dot{x}(t)^2\right\} = +\infty.$$

In [15], Lazer and Leach studied the nonlinear perturbations of a forced harmonic oscillator at resonance and provided a necessary and sufficient condition for the existence of harmonic solutions (defined in the next section). Specifically, they considered the following equation

$$\ddot{x} + n^2 x + \phi(x) = p(t), \tag{1}$$

where $n \in \mathbb{N}$, $\phi$ is a given continuous and bounded nonlinear function, and $p(t)$ is a given $2\pi$-periodic forcing. When

$$\left|\int_0^{2\pi} p(t)e^{-int}dt\right| < 2\left(\lim_{x\to+\infty} \inf \phi(x) - \lim_{x\to-\infty} \sup \phi(x)\right),$$

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*Key words and phrases.* Non-smooth dynamical systems; bounded solution; SD oscillator; harmonic solution; quasi-periodic solution.



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