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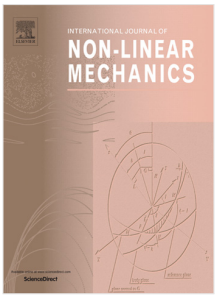

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Fifth order Camassa-Holm model with pseudo-peakons and multi-peakons

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Abstract

In this paper, we study a fifth order Camassa-Holm (FOCH) model with peakon solutions.Some interesting solutions are obtained including explicit single pseudo-peakons, two-peakon, and N-peakon solutions. In particular, detailed dynamical interactions for two-peakons and three-peakons are investigated numerically.

2000 Mathematics Subject Classification: 35C08, 37K40.

Keywords: Fifth order Camassa-Holm (FOCH) model, Pseudo-Peakons, Two-peakons, Three-peakons.

1 Introduction

Since 1993 [5], the Camassa-Holm (CH) equation has attracted much attention. This equation has a physics background with shallow water propagation, the bi-Hamiltonian structure, Lax pair, and explicit solutions including classical soliton, cuspon, and peakon solutions. In 1995, Calogero [6] generalized the CH type systems to a large class of mechanical models.Later, Ragnisco and Bruschi [21] and Suris [22] discussed the dynamics of the CH peakons in terms of an N-dimensional completely integrable system with providing Lax pair of the CH dynamical peakon and an $N \times N$ r-matrix structure [21].

In 1999, Degasperis and Procesi [9] extended the CH equation to the following water wave equation: $m_t + um_x + 3mu_x = 0,\ m = u - u_{xx}$, which is called the Degasperis and Procesi (DP) equation and also integrable with the peakon solutions. In spirit, the DP equation belongs to the b-family [13] $m_t + um_x + bmu_x = 0,\ m = u - u_{xx}$, where b is a real constant. In the b-family equations, the members with only $b = 2, 3$ are integrable [15], namely, the CH ($b = 2$) and the DP ($b = 3$) are the only integrable members in the b-family. In addition to the peakon solutions, the CH equation also possesses other type interesting solutions, including billiard solutions, piece-wise smooth solutions and algebro-geoemtric solutions [1, 2, 3, 4, 8, 19].

The DP equation has a 3×3 Lax representation with a third order differential spectral problem [10], which guarantees the integrability of the DP equation.It also has peakon and multi-peakon solutions [14]. In 2002, the DP equation was extended to an integrable hierarchy and dealt with its parametric solution and stationary solutions [18].

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