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Stochastic dynamic behavior of electrostatically actuated clamped–clamped microbeams with consideration of thermal field

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ABSTRACT

Environment perturbation and structure vibration can lead to the change of the thermal field, which may have certain effects on intrinsic frequency and dissipation mechanism for small-scale mechanical resonators. This article aims to theoretically investigate static and stochastic dynamic behavior of electrostatically actuated microbeams via a low dimensional model. Considering thermoelastic damping and thermal residual stress, an improved single degree of freedom model to describe microbeam-based resonators is obtained by using Hamilton's principle and Galerkin method. Through static behavior analysis, the influence of temperature field and electric field on the natural frequency and thermoelastic damping of the system is theoretically derived. The results show that the perturbation of environment temperature can significantly change the natural frequency and quality factor of the system. The following, a random frequency perturbation parameter is introduced to describe the perturbation of the environment temperature. An efficient approximation method is proposed to qualitatively study nonlinear stochastic dynamic behavior under small perturbation of stochastic parameter. It is found that the randomness of environment temperature suppresses nonlinear behavior and reduces the possibility of large amplitude vibration. Typically, the environment temperature disturbance reduces the resonance frequency of the system. Finally, the numerical method is put forward to not only verify the validity of the theoretical method but also quantitatively give the effects of noise strength and correlation rate on the nonlinear dynamic behavior. The present work provides theoretical framework for analyzing the effects of the perturbation of temperature field on nonlinear system response.

1. Introduction

Doubly clamped microbeams have been widely applied in many micro-electro-mechanical systems (MEMS) devices, such as energy harvester [1], microbeam resonator [2–4], gyroscope [5], sensor [6,7] and so on. Due to the existence of structure nonlinearity and nonlinear electrostatic force, they can exhibit rich nonlinear dynamic behaviors [8–10]. Besides, the microbeam resonator is in the multi physical field coupling environment [11]. With the improvement of MEMS performance, the influence of complex environment on MEMS dynamics has attracted more and more attentions. Environment perturbation and structure vibration can lead to the change of the thermal field, which may have certain effects on intrinsic frequency and dissipation mechanism for small-scale mechanical resonators.

Considering the fundamental frequency vibration, extensive studies mainly focus on the static and dynamic behavior of microbeam. Many researchers studied nonlinear dynamic behavior and pull-in instability which is always a key issue in the design of MEMS [12]. For instance, Younis et al. [13–19] traversed nonlinear dynamic behaviors of electrically actuated MEMS beams and arches. Galerkin method, Differential Quadrature method and Shooting method were introduced to investigate numerically static pull-in and dynamic pull-in phenomena. Han et al. [20] investigated the static and dynamic characteristics of a doubly clamped microbeam-based resonator driven by two electrodes and studied its dynamic pull-in. Krylov [21] proposed a largest Lyapunov exponent criterion and well evaluated the dynamic pull-in instability of a doubly clamped microbeam. Nonlinear model analysis was introduced to investigate the dynamics of a doubly clamped microswitch in the presence of geometric nonlinearity and nonlinear energy coupling [22]. In a word, a great deal of work has been done on the pull in instability and nonlinear dynamic behavior of microbeam systems. However, most of the above examples simplify or ignore the

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effects of environmental factors, which is not conducive to studying the influence of environmental interference on the system.

The thermoelastic damping is a fundamental dissipation mechanism caused by coupling of structure vibration and thermal field change, which is evaluated in light of recent efforts to design high-Q micrometer electromechanical systems [23,24]. A high quality factor can result in low power requirements, improved stability, and increased sensitivity for the system. It has been proved in the literature that thermoelastic damping is a significant dominant energy dissipation source at room temperature and vacuum in resonators [25–28]. Nayfeh and Younis [29,30] firstly investigated the thermoelastic damping in the microplates and utilized the perturbation method to derive an analytical expression for the quality factors of the microplates under electrostatic loading and residual stresses in terms of their structural mode shapes. Prabhakar et al. [31] investigated the thermoelastic damping in hollow, slotted, thin films and layered microresonators. Zhang et al. [32] described governing equations for modified coupled thermoelasticity in micro- and nanomechanical beam resonators and derived an analytical model of thermoelastic damping by using the complex-frequency approach. Abbas et al. [33] studied the generalized thermoelastic vibration of a bounded nanobeam resonator under fractional order theory of thermoelasticity. Analytical expressions for the detection, temperature change, frequency shift, and thermoelastic damping were derived for the beam. The above studies treated environmental temperature as a definite amount. In fact, the presence of environment perturbation can lead to uncertainty of the temperature field, which can affect thermoelastic damping and thermal residual stress. Some researchers introduced noise to deal with environmental disturbances. Verma et al. [34] presented a theory for stochastic resonance in MEMS capacitive chemical sensors and laid a framework for the calculation of sensor response. Due to different types of coupling between stress field and thermal field associated with the cantilever body material and the surrounding, the various noise mechanisms arise, which can lead to random modulation of the natural frequency. Gitterman [35] proposed an efficient method to obtain the approximate response of the system with uncertain frequency. Guo et al. [36] used the developed and improved exponential polynomial closure method to study nonlinear oscillators with random colored noise excitations and obtained the probabilistic solutions. Besides, the Fokker–Planck formalism is introduced to present a general analytical framework to address the stochastic dynamics [37,38]. The framework can be used to investigate stochastic resonance-type phenomena in coupled arrays of nonlinear oscillators.

It can be concluded from the above analysis that complex dynamic behaviors and oscillator performance are both important in the design of MEMS and should be taken into account [39,40]. Thermal residual stress and thermoelastic damping have important effects on the natural frequency and dynamic behavior of the system. The thermal field can affect thermal residual stress and thermoelastic damping, which may have certain effects on resonance frequency and dissipation mechanism for small-scale mechanical resonators. For the deterministic system, the researchers studied the influence of thermal residual stress and thermoelastic damping on the static and dynamic behavior of micromechanical systems [23]. However, the nonlinear electrostatic force and geometric nonlinearity can complicate the dynamics mechanism. Besides, the environment temperature disturbance can lead to uncertainty of the temperature field, which can affect thermoelastic damping and thermal residual stress. To the best of our knowledge, there are few qualitative results about a general analysis of static and stochastic dynamics behavior of electrostatically actuated clamped–clamped microbeams considering random thermal field, which motivates our present work. In this paper, we comprehensively study the influence of thermal field and electric field on the natural frequency and dissipation mechanism of the micromechanical system. Typically, we introduce random vibration theory to study the influence of stochastic temperature field on the dynamic response of the system. Here, a random frequency perturbation parameter is considered to describe the perturbation of the environment temperature. An efficient approximation method is proposed to qualitatively study nonlinear stochastic dynamic behavior under small perturbation of stochastic parameter. The work of this article may improve the performance of micromechanical resonators.

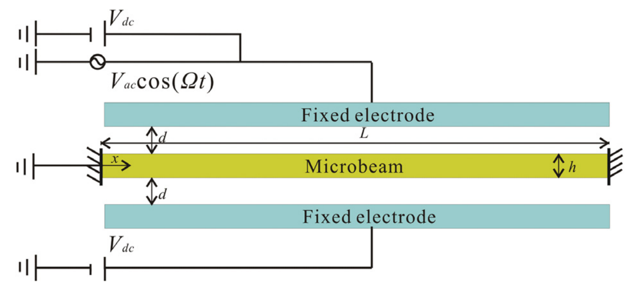

Fig. 1. Schematic of an electrically actuated microbeam.

The structure of this paper is as follows. In Section 2, the Hamilton's principle and the Galerkin discretization are applied to obtain a low dimensional model. Then static analysis is carried out under a DC voltage. In Section 3, an efficient approximation method is proposed to qualitatively study nonlinear stochastic dynamic behavior under small perturbation of stochastic parameter. In Section 4, we analyze stochastic dynamic behavior of the system by using the theoretical method. Meanwhile, the effects of noise strength and correlation rate on the oscillation amplitude are shown systematically. In Section 5, the numerical method is proposed to verify the validity of the theoretical method. Meanwhile, the effects of noise strength and correlation rate on the dynamic behavior are investigated. Finally, summary and conclusions are presented in the last section.

2. Mathematical model

2.1. Governing equation

As shown in Fig. 1, an electrically actuated microbeam is considered. The actuation of the microbeam is realized by means of two symmetric electrostatic bias voltages and an AC voltage component, which is widely used in microresonators [41]. Considering the influence of temperature field on the system, the modified thermoelastic strain and thermal residual stress are introduced. By using Hamilton's principle, the equation of motion that governs the transverse deflection $Y(x,t)$ is written as [9,23]

$$\rho A \frac{\partial^2 Y(x,t)}{\partial t^2} + \frac{\partial^2}{\partial x^2}[EI\frac{\partial^2 Y(x,t)}{\partial x^2} + E\alpha I_T] = [\frac{EA}{2L}\int_0^L (\frac{\partial Y(x,t)}{\partial x})^2 dx + N_T]\frac{\partial^2 Y(x,t)}{\partial x^2} + q \tag{1}$$

The boundary conditions are

$$Y(0,t) = \frac{\partial Y(0,t)}{\partial x} = Y(L,t) = \frac{\partial Y(L,t)}{\partial x} = 0 \tag{2}$$

The quantities I and $I_T = \int_A y\theta dydz$ are integrals over the cross section of the beam giving the mechanical and the thermal contributions to its moment of inertia [23]. The first term on the right hand of Eq. (1) represents mid-plane stretching effect. Here, $N_T = E\int_A \alpha\Delta T dydz$ is thermal residual stress, x is the position along the beam length, A is area of the cross section, α is the linear thermal-expansion coefficient, θ is relative temperature field caused by structural vibration [23], L is the length of beam, E is Young's modulus, t is time, ρ is the material density. The last term represents the parallel-plate electric actuation which is composed of DC and AC components

$$q = \frac{\varepsilon_0 b[V_{dc} + V_{ac}\cos(\Omega t)]^2}{2(d-Y)^2} - \frac{\varepsilon_0 b V_{dc}^2}{2(d+Y)^2} \tag{3}$$

where b is the microbeam width, d is the gap width, and ε_0 is the dielectric constant of the gap medium.

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