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Analytical model for deep bed filtration with multiple mechanisms of particle capture

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ABSTRACT

A model for deep bed filtration of a monodisperse suspension in a porous medium with multiple geometric particle capture mechanisms is considered. It is assumed that identical suspended particles can block pores of different sizes. The pores smaller than the particle size are clogged by single particles; if the pore size exceeds the diameter of the particles, it can be blocked by bridging — several particles forming various stable structures.

An exact solution is obtained for constant filtration coefficients. Exact solutions for non-constant filtration functions are obtained on the concentrations front of the suspended and retained particles and at the porous medium inlet. Asymptotic solutions are constructed near these lines. For small and close to constant filtration functions, global asymptotic solutions are obtained.

A basic model with two mechanisms of particle capture is studied in detail. Asymptotic solutions are compared to the results of numerical simulation. The applicability of various types of asymptotics is analyzed.

1. Introduction

Understanding the processes of transport and deposition of solid particles of suspensions and colloids in porous media is vital for the control of many engineering systems and in the study of natural structures. Deep bed filtration plays an important role in the contamination of groundwater and in the formation damage during oil production. Filtration makes possible separation of solid particles from a liquid mixture in chemical and biochemical industries, purification of drinking water, industrial waste disposal, groundwater remediation, decontamination of aquifers from viruses and bacteria [1–7].

A one-dimensional filtration problem for a monodisperse suspension in a homogeneous porous medium is considered. The fluid flow transports solid particles through pores. Some particles are retained on the porous medium frame and form a deposit. If the particle diameter exceeds the pore size, the deposit at the inlet of the porous medium forms a filter cake. Usually a porous medium contains pores of various sizes, from pores smaller than the particle diameter up to large pores whose sizes are several times larger than the particles. The particles penetrate deep into the porous medium and get stuck over the entire length of its frame. This process is called deep bed filtration [8–11]. To study the filtration of suspensions and colloids in a porous medium, various models are used that take into account the different causes of particle transport and retention [12–15]. In many cases the main mechanism for particles retention is size-exclusion: single particles pass freely through large pores and get stuck at the inlets of small pores (Fig. 1a, b). The filtration model for a monodisperse suspension with size-exclusion was studied in [16–18].

A one-dimensional model for the filtration of a monodisperse suspension in a porous medium includes the equation for the mass balance of suspended and retained particles and the kinetic equation for the growth of the retained particles concentration. In the domain

$$\Omega = \{(x,t) : 0 < x < 1, t > 0\}$$

the standard macroscopic dimensionless equations have the form

$$\frac{\partial C}{\partial t} + \frac{\partial C}{\partial x} + \frac{\partial S}{\partial t} = 0; \tag{1}$$

$$\frac{\partial S}{\partial t} = \Lambda(S)C; \tag{2}$$

where $C(x,t)$; $S(x,t)$ are the volumetric concentrations of suspended and retained particles.

The boundary and initial conditions for the system (1), (2) are

$$C|_{x=0} = 1; \tag{3}$$

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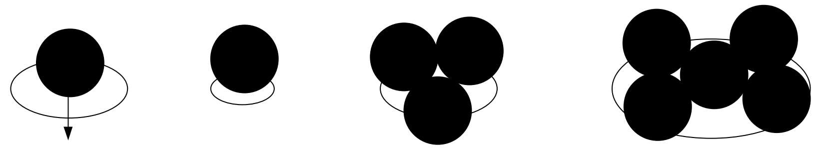

Fig. 1. Particle transport and pore blocking by separate particles and stable particle bridges.

$$C|_{t=0} = 0;\ S|_{t=0} = 0. \tag{4}$$

For the problem (1)–(4) the exact analytical solution can be obtained for any form of the filtration function $\Lambda(S)$ [19].

The filtration function $\Lambda(S)$ which depends on the pore geometry and on the particle and rock properties is determined experimentally [20]. For low retention concentration, $\Lambda(S)$ is constant and is called the filtration coefficient; it can be determined using either the breakthrough concentration or the pressure drop growth during the clean-bed suspension injection [1,21]. In the case of size-exclusion (blocking of pores of the small size by the single particles) the filtration function is proportional to the concentration vacancies and is called the blocking (Langmuir) function

$$\Lambda(S) = \lambda\left(1 - \frac{S}{S_{\max}}\right).$$

In the general case the filtration function $\Lambda(S)$ is non-linear; it can be determined from the time of the breakthrough concentration. The corresponding inverse problem is ill-posed and requires regularization [22]. Yet, the problem of determination of the filtration function using the retention profile is a well-posed inverse problem [23]. However, the breakthrough concentration depending on time is measured by particle counting and is readily available from laboratory core floods [24,25]. On the contrary, the retention profile is determined by complex and cumbersome CT technique [26,27]. A detailed description of laboratory core floods of suspensions and colloids can be found in [28].

Usually, the existence of an analytical solution of the direct problem significantly simplifies the solution of the inverse problem [1,29].

The retention rate given by Eq. (2) is proportional to the suspended particles concentration C only for low suspension concentrations; for large concentrations, it is proportional to $f(C)$ which is called the suspension function.

Experiments show that pores larger than the particle diameter can be clogged by several particles. Near the pore inlet at a sufficiently high velocity of the carrier fluid, the hydrodynamic characteristics of the flow change, and the forces of the suspended particles attraction begin to exceed the forces of mutual repulsion. As a result, complex multi-particle structures are formed directly in the fluid and approach the pores as single units. At the pore inlet, such a structure either collapses and falls into the pore or docks to the pore inlet and blocks it (Fig. 1c, d). This phenomenon is called bridging [30,31].

In the above works, only one of the possible mechanisms of pore blocking is considered: either bridging or size-exclusion. In [32], a model of deep bed filtration with several particle capture mechanisms is presented. However, it only considers the capture of individual particles, rather than the formation of arched bridges of several particles. For several equations of the form (2) with different filtration functions, the retention rate is proportional to the concentration of suspended particles. Nevertheless, the multi-particle bridging is one of the most significant capture mechanisms [1]. Detailed results of experiments with size-exclusion and bridging acting simultaneously are given in [33]. No analytical models for multi-particle bridging are available in the literature.

The present paper fills the gap. We consider a filtration model in which several particle capture mechanisms operate simultaneously with arched bridges of different configurations. It is assumed that the bridge configuration for pore blocking is determined by the ratio of the particle and pore sizes. Pores with sizes smaller than the particle size are clogged by single particles. If the pore size exceeds the particle size it can be blocked by several particles which form a stable structure of an arched bridge at the pore inlet. Suppose that there are several different types of stable arched bridges that can be formed at the inlets of pores whose sizes exceed the suspended particle size (for example, Fig. 1c, d); the bridge that blocks the pore cannot be destroyed by the fluid flow or by the other particles.

The structure of the text is as follows. In Section 2, a mathematical model for the filtration with several particle capture mechanisms is developed and the main properties of the solution are studied. The exact solutions to the problem are obtained in Section 3. In Sections 4 and 5 local and global asymptotic solutions are constructed. The results of numerical simulation for the basic model with two particle capture mechanisms are given in Section 6. Discussion and Conclusions in Sections 7, 8 complete the paper.

2. A mathematical model and the main properties of the solution

In the proposed model, the kinetic equation of deposit growth in pores of a certain size depends on the type of the arched bridge. Assume that during the filtration n stable bridges consisting of $k_1, k_2, \ldots, k_n$ particles can be formed blocking pores of various sizes, and one large pore can be clogged by a bridge of only one specific configuration with a given number of particles. In this case the rate of deposit bridge growth $\partial S_i/\partial t$ is proportional to the bridging function $F_i(C)$ that depends nonlinearly on the suspended particles concentration. The filtration function $\Lambda_i(S_i)$ depends on the deposit bridge concentration S_i.

In the modified filtration model the equation of mass balance includes the total deposit S which usually is equal to the sum of the partial deposits S_i. Consider a more general case in which the total deposit is a linear combination of partial deposits.

In the domain Ω the model of the one-dimensional filtration problem with various geometric mechanisms of particle capture is determined by the equations

$$\frac{\partial C}{\partial t} + \frac{\partial C}{\partial x} + \frac{\partial S}{\partial t} = 0; \tag{5}$$

$$\frac{\partial S_i}{\partial t} = \Lambda_i(S_i)F_i(C), \quad i = 1, 2, \ldots, n; \tag{6}$$

$$S = \alpha_1 S_1 + \alpha_2 S_2 + \cdots + \alpha_n S_n; \tag{7}$$

where $C(x,t)$; $S_i(x,t)$ are the concentrations of suspended particles and of partial bridging deposits, $\alpha_1, \alpha_2, \ldots, \alpha_n$ — positive constants.

Note that size-exclusion clogging of small pores by single particles is one of the particle capture mechanisms. To include this mechanism into the model one should assume that $k_1 = 1$; $F_1(C) = C$.

Assume that a suspension with suspended particles of constant concentration is injected at the porous medium inlet:

$$C|_{x=0} = 1; \tag{8}$$

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