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Size effects in wedge indentation predicted by a gradient-enhanced crystal-plasticity model

M.J. Lewandowski, S. Stupkiewicz*

Institute of Fundamental Technological Research, Polish Academy of Sciences, Pawińskiego 5b, 02-106 Warsaw, Poland

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ABSTRACT

A recently developed gradient-enhanced crystal-plasticity model is applied to predict the size effects in wedge indentation. In the model, the internal length scale is defined through standard quantities that appear in the underlying non-gradient hardening law. A careful calibration of the non-gradient hardening law is thus performed, and the model is validated against published experimental results. To this end, a comprehensive computational study of wedge indentation into a nickel single crystal is performed, and the obtained results show a good agreement with the experiment in terms of the load–penetration depth curves for three wedge angles, as well as in terms of the distributions of lattice rotation, GND density, and net Burgers vector. For the indentation depth of about 200 μm, as employed in the experiment, the predicted size effects are insignificant. Accordingly, the size effects are next studied for the indentation depth varied between 200 μm and 1 μm. As an intermediate result, apparently not published to date, the general 3D crystal plasticity model with anisotropic hardening is consistently reduced to a 2D plane-strain model in which plastic deformation is realized by three effective in-plane slip systems, each representing two crystallographic slip systems.

1. Introduction

A considerable interest in the materials science and mechanics communities is currently focused on size effects in metal plasticity. Size effects induced by strain gradients are observed in micro-torsion (Fleck et al., 1994), micro-bending (Stölken and Evans, 1998) and micro/nano-indentation (Ma and Clarke, 1995; McElhaney et al., 1998; Pharr et al., 2010). It is commonly agreed that the related hardening mechanism ('smaller is stronger') is associated with the geometrically necessary dislocations (GNDs) that accommodate the strain gradients (Nye, 1953; Ashby, 1970). The strain gradients increase with decreasing sample size or indentation depth, and the additional GND hardening is then responsible for the related size effects. Size effects can also be observed in the absence of strain gradients, e.g., due to dislocation starvation in small samples (Greer and Nix, 2006). Those effects are, however, not considered in this work.

Instrumented indentation is a highly versatile and popular testing technique, and hence the indentation size effect is by far the most frequently studied size effect. For geometrically self-similar indenters, like pyramids and cones, the usual size effect manifests itself in the increase of hardness, defined as the ratio of the load to the projected contact area, with decreasing indentation depth (Pharr et al., 2010). The indentation size effect is also characteristic for spherical indentation (Swadener et al., 2002), where the hardness increases with decreasing indenter radius (for a fixed ratio of the indentation depth to the indenter radius).

Size-dependent response has also been observed in wedge indentation. Chen et al. (2012) performed indentation of an aluminum

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[*] Corresponding author.
E-mail address: sstupkie@ippt.pan.pl (S. Stupkiewicz).

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single crystal by a series of diamond wedges and reported a significant increase of hardness for the indentation depth below 1–2 μm, while the response was essentially size-independent for the indentation depth above 5 μm. Notably, the wedge indentation technique is definitely less popular than the usual Berkovich or spherical tip indentation. The work of Chen et al. (2012) is apparently the only one showing the size effects in wedge indentation experimentally.

In wedge indentation, an adequately oriented single crystal is deformed in (approximately) plane-strain conditions. This offers significant benefits for both experiment and modelling, as discussed below. A wedge indentation technique accompanied by a detailed characterization of lattice rotations using electron backscatter diffraction (EBSD) has been developed by Kysar et al. (2007, 2010). In this technique, the specimen midplane is exposed after indentation, and the in-plane rotations near the indent are measured with a high resolution. In the experiment, the indentation depth was close to 200 μm, which facilitated high-resolution EBSD measurements. However, as the indentation depth was relatively large, the overall response is not expected to be affected by size effects. Further processing of the measured rotation field delivers additional valuable data such as the spatial distributions of the total and slip-system-resolved GND densities (Kysar et al., 2010) and the magnitude and orientation of the net Burgers vector density (Sarac et al., 2016). The technique is quite unique to the wedge indentation because it relies on the plane-strain assumption. Note that full-field measurements of lattice rotations in the vicinity of three-dimensional indents are readily available (Zaafarani et al., 2006; Rester et al., 2008). However, more involved techniques are then needed to estimate the GND density, and the achievable resolution is lower (Demir et al., 2009; Wilkinson and Randman, 2010).

From the modelling point of view, with the focus on crystal plasticity, the wedge indentation is also quite advantageous because the problem can be formulated as a two-dimensional (2D) one thus offering a significant reduction of the computational cost compared to the three-dimensional (3D) setting in the usual Berkovich or spherical tip indentation. For a face-centred cubic (fcc) crystal deformed within the (1 1 0) plane, the crystal plasticity model itself can also be simplified by introducing three effective, composite in-plane slip systems, each representing two crystallographic slip systems (Rice, 1987). This feature has actually been exploited in the wedge indentation experiments of Kysar et al. (2007, 2010). The crystal and the wedge were adequately oriented such that the plane-strain deformation with three composite slip systems could be assumed in the processing of the EBSD measurements. The reduction of the number of slip systems in an fcc crystal from 12 to 3 becomes particularly advantageous when it comes to the modelling of size effects using a gradient crystal-plasticity model. Indeed, in many formulations of gradient crystal-plasticity (including the one used in this work), the slip rates on individual slip systems (or related variables, e.g., the dislocation densities on individual slip systems) constitute the global unknowns in a boundary value problem. For an fcc crystal, the 3D model involves thus 15 global unknowns (3 displacements and 12 slip rates), while the reduced 2D model involves 5 global unknowns (2 displacements and 3 slip rates). The saving in the computational cost associated with the transition from 3D to 2D is thus substantial.

In the case of the classical (size-independent) crystal plasticity, full 3D simulation of indentation does not constitute a major problem at the current stage of development of computational methods. An overview of the related literature can be found in Petryk et al. (2017) along with a methodology for estimating the strain hardening exponent based on the analysis of the pile-up/sink-in pattern around the residual impression after spherical indentation.

However, modelling of the indentation size effect is in most cases limited to isotropic plasticity (e.g., Huang et al., 2000, 2006; Qu et al., 2006), often based on a simplified geometrical model (e.g., Nix and Gao, 1998; Abu Al-Rub, 2007). Simulations based on crystal plasticity with an internal length scale are much more scarce, even if a considerable number of gradient crystal-plasticity models are available in the literature, for instance, those of Gurtin (2000), Forest et al. (2002), Evers et al. (2004), Han et al. (2005), Kuroda and Tvergaard (2008), Bargmann et al. (2011), Hochrainer et al. (2014), Anand et al. (2015), Wulfinghoff et al. (2015), and Kratochvil and Kruzik (2016), to mention just a few representative examples. To the best of our knowledge, the only 3D gradient crystal-plasticity simulations of indentation are those of Lee and Chen (2010) and Gao et al. (2015), in each case employing a version of the so-called conventional mechanism-based strain-gradient (crystal) plasticity theory (Huang et al., 2004; Han et al., 2005), and the recent simulations by Stupkiewicz and Petryk (2016), which employ the gradient-enhanced crystal-plasticity model developed by Petryk and Stupkiewicz (2016).

The model of Petryk and Stupkiewicz (2016) is also employed in the present work. In this model, the classical framework of crystal plasticity (Hill, 1966; Rice, 1971; Hill and Rice, 1972) is enhanced with slip-rate gradient effects by extending the usual anisotropic hardening law with a single isotropic term that represents the GND hardening. In contrast to the frequently used split of the total dislocation density into the densities of statistically stored dislocations (SSDs) and GNDs (e.g., Ashby, 1970; Fleck and Hutchinson, 1993; Nix and Gao, 1998), the model employs such a split applied in an incremental form only. This apparently minor difference has a significant influence on the resulting model. The internal length scale, which is derived in a closed form and depends on the current flow stress and hardening rate, is shown to be closely related to the mean free path of dislocations and thus possesses a direct physical interpretation. The resulting ‘minimal’ gradient enhancement of the hardening law is free of any fitting parameters. The computational treatment of the model relies on element-scale averaging of local slip rates, and the resulting non-local slip rates are used to compute the slip-rate gradients that govern the GND hardening. The averaging involves an independent length-scale parameter of purely numerical (or regularization) nature. Spherical indentation into a copper single crystal has been simulated, and the predicted dependence of hardness on the indenter radius shows a good agreement with experiment (cf., Stupkiewicz and Petryk, 2016).

In this paper, the gradient-enhanced crystal plasticity model of Petryk and Stupkiewicz (2016) is applied to predict the size effects in wedge indentation. Specifically, the effect of the indentation depth on the hardness, residual imprint and sink-in, lattice rotation and GND distribution is examined for the wedge indentation into a nickel single crystal. To the best of our knowledge, results of such scope have not been reported so far. Apparently, the only related simulations of wedge indentation are those of Reuber et al. (2014) and Bittencourt (2018). Reuber et al. (2014) used a nonlocal crystal plasticity model that includes dislocation transport. With