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About an approach to the determination of the critical time of viscoelastic functionally graded cylindrical shells

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ABSTRACT

In this study the dynamic stability of viscoelastic functionally graded cylindrical shells (VEFGCSs) under an axial load with different initial conditions is investigated. Mathematical models are constructed for the problem of dynamic stability of the VEFGCSs, which is characterized simultaneously by taking into account both viscoelastic and FGM features. The basic equations of VEFGCSs are described by integro-differential equations using the linear viscoelasticity theory. An approach is developed to the determination of the critical times (CTs) for VEFGCSs with different initial conditions. Finally, the numerical analyzes are performed to demonstrate the influences of the initial conditions, the FGM profiles and the rheological parameter on the critical times for various geometric characteristics of the cylindrical shells.

1. Introduction

The main problems in the field of science and technology are related to the analysis of the vibration and stability of viscoelastic structures because of their wide range of applications. In the mathematical modeling of processes occurring in viscoelastic materials, there is a so-called memory system whose behavior depends on the entire history of this system, which is not yet fully defined and therefore describes the integro-differential equation as a function of time. In the studies, the inherited theory of viscoelasticity, constructed on the basis of the Boltzmann superposition principle, is used as a theory describing the time-dependent processes of deformation. For this reason, a number of fundamental research and monographs devoted to the theory of creep and the theory of viscoelasticity have been published [1–7]. When calculating structures for strength and stability, it is essential to take into account the viscoelastic properties of deformable bodies, which leads to the appearance of additional integral terms in the differential equations. In light of basic information on the viscoelastic materials, many publications on the vibration and stability of structural elements consisting of these materials have been published, from the first period of research to the present day. Among these publications there are some precious publications [8–25].

The FGMs belong to the class of special composite materials obtained by combining two or more composite phases with continuous distribution in the preferred orientation. The changes in the composition of the components lead to an inhomogeneous microstructure that causes gradual changes in the properties of the macroscopic material. This important property can improve the thermal properties, eliminate the stress singularity, increase the fracture toughness and prevent stratification with the conventional composites. The gradual changes in the properties of the material make it possible to obtain the desired properties for various applications. Advantages of FGM in comparison with traditional materials show that they have a great potential in aerospace structures, nuclear facilities, medical implants and many other fields of engineering [26–31]. Recently, some remarkable studies have been performed describing the distribution of volume fractions and features of FGMs [32–37]. FGMs can be ideal when operating conditions are severe, for example, components of thermal engines or thermal shields of missiles. Under these conditions, FGMs may exhibit time-dependent behavior. Full use of the potential of FGM requires the development of appropriate methodology modeling. A simple but realistic phenomenological model for describing the time-dependent behavior is the linear viscoelasticity [38]. In recent years, some studies have been carried out on the theory of viscoelasticity for FG viscoelastic structural elements, among which we note papers [39–44].

Some studies have been carried out in recent years about the behavior of plates and shells made of viscoelastic FG materials. Within these studies, the number of publications devoted to behavior of the VEFGCSs is limited. Mao et al. [45] studied the flexural creep and post-buckling analysis of laminated piezoelectric viscoelastic FG plates (VEFGPs) taking into account the SDT and the geometric nonlinearity of von Karman. Shariyat and Alipor [46] developed a series of solutions for free vibration and damping analyzes of VEFGPs with variable thickness on the elastic bases. Shariyat and Nasab [47] studied the low-

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speed effect of general VEFGPs, taking into account the determination of the apparent stiffness of the contact area and the change in the Hertz contact law. Barretta et al. [48] investigated torsion of FG nonlocal viscoelastic circular nanobeams and developed an effective solution procedure based on the Laplace transformation, providing a new correspondence principle in nonlocal viscoelasticity for FGMs. Yang et al. [49] provided a unified, but accurate solution for the analysis of vibration and damping of VEFGPs with arbitrary boundary conditions. Deng et al. [50] developed a hybrid method that combines the reverberation ray array method and the wave propagation method to study the stability of multi-span, viscoelastic FGM tubes transporting liquid. Paland and Alibeiglo [51] investigated the static and vibrational analysis of a sandwich-cylindrical shell with a FG core and a viscoelastic interface using DQM. Swain and Roy [52] formulated an eight-node shell element with five degrees of freedom for a node to study the damping characteristics of the oscillations of spherical shells made by CNT-CFRP-2DWF composites, based on the SDT in accordance with the Mindlin hypothesis and Koiter shell theory. Sofiyev [53] presented the solution of the dynamic stability of heterogeneous orthotropic viscoelastic cylindrical shells based on the CST.

Most of the above studies are based on numerical methods, their use is more labor-intensive, and they are also less effective for visualization for the determination of the critical time. The objective of the present work is to the study dynamic stability of VEFGCSs under an axial load, with two initial conditions. The basic equations of VEFGCSs are described by integro-differential equations using the linear viscoelasticity theory. An analytical approach is developed to determination of the critical time for VEFGCSs. Finally, are used the various FGM profiles to demonstrate the influences of material gradient, initial conditions, shell characteristics and rheological parameter on the CT.

2. Mathematical modeling of the problem

2.1. Material constitutive relations

We shall assume that the VEFGCS with the radius R, length L and thickness h, subjected to a uniform axial compression of magnitude P, which is less than the elastic critical load P_{cr}. (See, Fig. 1). We shall use the orthogonal system of coordinates usual for cylindrical shells; x_1, x_2 and x_3 axes are in the axial, circumferential, and inward radial directions, respectively. We shall designate the displacements; u_1 is the direction of the generatrix, u_2 is a circular displacement, and u_3 is a radial displacement. In addition, we assume that the stresses acting normally to the reference surface are negligibly small in comparison with the other components.

The volume fractions of FGMs are defined by the power-law function, as [31]:

$$V_c(X_3) = (X_3 + 0.5)^d,\ V_c(X_3) + V_m(X_3) = 1,\ X_3 = x_3/h \tag{1}$$

where $V_m(X_3)$ and $V_c(X_3)$ are the volume fractions of the metal and the

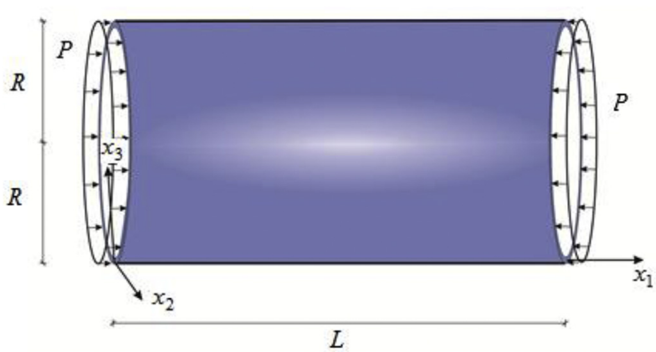

Fig. 1. The VEFGCS subjected to an axial load and notations.

ceramics phases, respectively, d is the volume fraction index, $0 \le d \le \infty$.

Pitakthapanaphong and Busso [28] proposed an inverse quadratic form for the volume fractions of the FGM as

$$V_c(X_3) = 1 - (0.5 - X_3)^2 \tag{2}$$

The effective material properties F_f are defined as [31].

$$F_f = F_0 T^{-1} F_{-1} + F_0 + F_0 T F_1 + F_0 T^2 F_2 + F_0 T^3 F_3 \tag{3}$$

Where $F_j,\ j = -1,\ 0,\ 1,\ 2,\ 3$ are the coefficients of temperature T (K) and are unique to the constituent materials.

The effective properties of FGMs are usually taken according to the rule of the mixture of materials and are expressed as follows:

$$F = F_m V_m(X_3) + F_c V_c(X_3) \tag{4}$$

where F, F_m and F_c are the generic material property, the properties of metal and ceramics, respectively [31,32].

According to Eqs. (1) and (4), the effective Young's modulus, Poisson's ratio and density of FGMs are obtained as follows:

$$E_{fg}(X_3) = (E_c - E_m)V_c(X_3) + E_m, \qquad \nu_{fg}(X_3) = (\nu_c - \nu_m)V_c(X_3) + \nu_m,$$
$$\rho_{fg}(X_3) = (\rho_c - \rho_m)V_c(X_3) + \rho_m \tag{5}$$

where E_m, E_c , ν_m, ν_c and ρ_m, ρ_c are the Young's modulus, Poisson's ratio and density of the metal and ceramic surfaces of FGMs, respectively.

From Eq. (5) it can be seen that the effective material properties of the ceramic-rich shells, when $V_c(X_3) = 1$ and the effective material properties of metal-rich shells, when $V_c(X_3) = 0$ are obtained. Detailed explanations and interpretations of FG profiles are presented in the study of Tornabene et al. [29,32,33,35,36].

Jin and Batra [30] proposed a different kind of FGMs that the material properties continuously change along the thickness directions according to the exponential law:

$$E_{\rm fg}(X_3) = E_m e^{(X_3+0.5)\ln(E_c/E_{\rm m})}, \qquad \nu_{\rm fg}(X_3) = \nu_{\rm m} e^{(X_3+0.5)\ln(\nu_c/\nu_{\rm m})},$$
$$\rho_{\rm fg}(X_3) = \rho_{\rm m} e^{(X_3+0.5)\ln(\nu_c/\nu_{\rm m})} \tag{6}$$

2.2. Basic relations and equations

In the given coordinate system, the constitutive relations of the VEFGCSs based on the linear theory of viscoelasticity can be presented in the form [4,33,39]:

$$\sigma_{x_1} = \frac{E_{fg}}{1-\nu_{fg}^2}\left\{\varepsilon_{x_1}^0 + \nu_{fg}\,\varepsilon_{x_2}^0 - x_3\left(\frac{\partial^2 u_3}{\partial x_1^2} + \nu_{fg}\frac{\partial^2 u_3}{\partial x_2^2}\right) - \int_0^t \left[\varepsilon_{x_1}^0 + \nu_{fg}\,\varepsilon_{x_2}^0 - x_3\left(\frac{\partial^2 u_3}{\partial x_1^2} + \nu_{fg}\frac{\partial^2 u_3}{\partial x_2^2}\right)\right]R(t-\tau)d\tau\right\}$$
$$\sigma_{x_2} = \frac{E_{fg}}{1-\nu_{fg}^2}\left\{\varepsilon_{x_2}^0 + \nu_{fg}\,\varepsilon_{x_1}^0 - x_3\left(\frac{\partial^2 u_3}{\partial x_2^2} + \nu_{fg}\frac{\partial^2 u_3}{\partial x_1^2}\right) - \int_0^t \left[\varepsilon_{x_2}^0 + \nu_{fg}\,\varepsilon_{x_1}^0 - x_3\left(\frac{\partial^2 u_3}{\partial x_2^2} + \nu_{fg}\frac{\partial^2 u_3}{\partial x_1^2}\right)\right]R(t-\tau)d\tau\right\}$$
$$\sigma_{x_1x_2} = \frac{E_{fg}}{2(1+\nu_{fg})}\left[\gamma_{x_1x_2}^0 - 2x_3\frac{\partial^2 u_3}{\partial x_1\partial x_2} - \int_0^t \left[\gamma_{x_1x_2}^0 - 2x_3\frac{\partial^2 u_3}{\partial x_1\partial x_2}\right]R(t-\tau)d\tau\right] \tag{7}$$

where $\varepsilon_{x_1}^0$, $\varepsilon_{x_2}^0$, $\gamma_{x_1x_2}^0$ are the strains in the reference surface and $R(t-\tau)$ is the relaxation kernel of the integral operator.

The stresses resultants and the stress couples are defined from the following relations [4]:

$$(n_{ij},\ m_{ij}) = \int_{-h/2}^{h/2} \sigma_{ij}[1,\ x_3]dx_3, \qquad (i,j) = (x_1,\ x_2) \tag{8}$$