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A study on the role of the initial conditions and the nonlinear dissipation in the non-Hermitian effective Hamiltonian approach

Santiago Echeverri-Arteaga[a], Herbert Vinck-Posada[a], Edgar A. Gómez[b,*]

[a] Departamento de Física, Universidad Nacional de Colombia, 111321 Bogotá, Colombia
[b] Programa de Física, Universidad del Quindío, 630004 Armenia, Colombia

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ABSTRACT

This work examines the initial conditions dependence and the influence of nonlinear dissipation in the quantum dynamics originated by the non-Hermitian effective Hamiltonian approach. In particular, we test the performance of this theoretical approach for modeling the dynamics of the pumped-dissipative Jaynes Cummings model under different initial states such as a Fock, Gaussian and maximally mixed states, and as well as for a polaritonic light-matter state. It is demonstrated unequivocally that whereas the non-Hermitian effective Hamiltonian approach fails in to reproduce correctly the time evolution of the density matrix elements and observables of the system, the corrected version of this theoretical approach recently developed by us shows an excellent agreement with simulations based on the Lindblad master equation formalism.

1. Introduction

The influence of the environment on the quantum system has received much attention over the last three decades, and a variety of theoretical approaches have been developed to investigate this fascinating issue. In this scenario, the non-Hermitian effective Hamiltonian (NHEH) approach has become an important methodology for studying dissipative quantum system, as well as the effect of many others decoherent processes which are included typically as non-Hermitian Hamiltonian terms. The NHEH approach has applications in fields such as in atomic and nuclear physics for studying shell models [1,2] and, moreover, of interesting and important applications in both molecular and quantum chemistry [3,4]. In recent years there has been considerable interest in this methodology and studies on many-body quantum systems made of ultracold atoms loaded into periodic lattice structures [5,6] have shown that, it is possible to identify stable quantum states as well as long-range coherent dynamics even in the dissipative case. Some studies based on this theoretical approach have been focused on the understanding and modelling of quantum driven-dissipative systems [7], critical behavior and phase transitions in quantum many-body systems [8,9]. In fact, the last few years have seen a growing interest in adapting the NHEH approach to both Markovian and non-Markovian master equations [10,11], and in dissipative quantum systems in which the adiabatic approximation can be carried out [12]. These theoretical efforts have also opened the door to many new and interesting research directions, and in particular some of them have been focused on the comparison and unification of non-Hermitian and Lindblad approaches with applications to open quantum optical systems [13,14]. Interestingly enough is an hybrid formalism that combines the characteristics of both of these approaches, and it has found applications in areas such as quantum fine-grained entropy in systems that are described by non-Hermitian Hamiltonians [15]. Also, for describing time-dependent correlation functions in systems whose dynamics are governed by non-Hermitian Hamiltonians of general type [16]. The NHEH

* Corresponding author.
E-mail address: eagomez@uniquindio.edu.co (E.A. Gómez).

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approach has been recognized as a powerful tool for approximating the dynamics of the open quantum systems, since this approach has several advantages from a computational point of view and it is well suited for stochastic wavefunction or quantum trajectory methods [17,18]. Surprisingly, despite a vast amount of theoretical works based on the NHEH approach, few is known about the reliability of this approach when it is applied directly for solving the dynamics of an open quantum system. In fact, this gap in the literature has called our attention and it has motivated a recent study on the performance of the NHEH approach for describing the dissipative quantum dynamics, and in particular it was concluded that this theoretical approach describes inadequacy the observables of the system [19]. Also, in this work we have developed an interesting methodology called the corrected NHEH approach [19] to overcome the deficiencies found in the traditional approach. We believe that additional studies showing a comparison between the traditional NHEH approach and our corrected NHEH approach will help to establish the reliability of this methodology, as well as to be highly regarded by scientists working in different research fields. The aim of this study is twofold: on the one hand, to demonstrate that the quantum dynamics originated by the NHEH approach in a well-known open quantum system of the cavity quantum electrodynamics (cQED) depends strongly on its initial conditions, and that it fails in general. On the other hand, to test the performance of the corrected NHEH approach under different initial conditions, as well as under more demanding decoherent processes as is the nonlinear dissipation and an incoherent pump in the system. This paper is organized as follows. In Section 2 we consider a two-level system interacting with an electromagnetic cavity mode in contact with a dissipative environment, and the NHEH approach is considered to describe cavity losses and spontaneous emission in the system. Moreover, the initial condition dependence on the NHEH approach is demonstrated through the time evolution of four particular initial quantum states. It is, by considering a Fock, Gaussian and maximally mixed states, as well as a polaritonic light-matter state. In this section also, the corrected NHEH approach is tested by means of various numerical calculations demonstrating the efficacy of the presented method. In Section 3 we extend our study to a more demanding situation, specifically, where there are nonlinear dissipation and incoherent pump in the system through non-Hermitian Hamiltonian terms. Furthermore, in this section, we test the performance as well as the initial condition dependence on the corrected NHEH approach. Our numerical results demonstrate that the corrected version of the NHEH approach is in excellent agreement with the predictions of the Lindblad master equation formalism. Finally, we conclude in Section 4.

2. Non-Hermitian effective Hamiltonian approach with linear dissipation but without pump.

Within the framework of cavity quantum electrodynamics (cQED), the most simple and interesting quantum model that studies the interaction between light and matter is the Jaynes-Cummings (JC) model. This theoretical model describes a two-level system (TLS) interacting with an electromagnetic cavity mode that in the rotating wave approximation its Hamiltonian is given by ($\hbar = 1$)

$$\hat{H}_{\rm JC} = \omega_x \hat{\sigma}^\dagger \hat{\sigma} + \omega_c \hat{a}^\dagger \hat{a} + g(\hat{\sigma}\hat{a}^\dagger + \hat{a}\hat{\sigma}^\dagger), \tag{1}$$

where g is the light-matter interaction constant between the cavity mode and the TLS. Furthermore, ω_x and ω_c are the frequencies associated to the TLS and the cavity mode, respectively. In this model $\hat{a}$ and $\hat{\sigma} = |G\rangle\langle X|$ are the annihilation and lowering operators for the cavity mode and the TLS, such that the action of the lowering operator on the excited state $|X\rangle$ leads to the ground state $|G\rangle$. Interestingly enough, is that the JC model has the conserved quantity known as number of excitations through the operator $\hat{N} = \hat{a}^\dagger\hat{a} + \hat{\sigma}^\dagger\hat{\sigma}$ that is diagonal in the bare-states basis $\{|\alpha, n\rangle \equiv |\alpha\rangle|_{\alpha=G}^{X} \otimes \ |n\rangle|_{n=0}^{\infty}\}$. In this basis of states, n and α corresponds to the number of photons in the cavity and one of the two possibles states of the TLS, respectively. Since the operator $\hat{N}$ defines the conserved quantity it is possible to study the quantum dynamics of the system within separate subspaces of the full state-space. In fact, each subspace is characterized by the eigenvalue n of the operator $\hat{N}$ and it is called the nth rung in the JC ladder of states. Taking into account that the NHEH approach are among the most widely used methods for incorporating dissipative effects in quantum systems, we incorporate two irreversible processes as the leakage of photons from the cavity at rate κ and the spontaneous emission at rate γ_x for describing the quantum dynamics of the dissipative JC model as follows:

$$\frac{d\hat{\rho}}{\rm dt} = \mathcal{H}_{\hat{H}}(\hat{\rho}^{[n]}), \tag{2}$$

where we have defined the superoperator $\mathcal{H}_{\hat{X}}(\cdot) = i(\cdot\hat{X}^\dagger - \hat{X}\cdot)$ and the operator $\hat{H} = \hat{H}_{\rm JC} + \hat{H}_{\rm eff}$. Notice also that the non-Hermitian effective term reads explicitly

$$\hat{H}_{\rm eff} = -\frac{i}{2}\kappa\hat{a}^\dagger\hat{a} - \frac{i}{2}\gamma_x\hat{\sigma}^\dagger\hat{\sigma}, \tag{3}$$

where $\hat{\rho}^{[n]}$ defines the density operator at the nth rung. It is worth to mention that this theoretical approach has been considered in the past for studying the quantum dynamics for an initially coherent-state field in a damped JC model [20]. In order to examine the initial condition dependence on both the dynamics and the observables of the system, let us consider the numerical solution of Eq. (2) under four different initial conditions or quantum states. Namely, the Fock state given by

$$\hat{\rho}_F(0) = |{\rm Gn}\rangle\langle{\rm Gn}| \tag{4}$$

with $n = 6$. The Gaussian and Maximally mixed states are given by

$$\hat{\rho}_G(0) = \sum_{n'=0}^{N} \frac{1}{\delta\sqrt{2\pi}} e^{-\frac{(\hat{a}^\dagger\hat{a} - \bar{n}\hat{I})^2}{2\delta^2}} |{\rm Gn'}\rangle\langle{\rm Gn'}| \tag{5}$$

and