Cement and Concrete Research xxx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

Cement and Concrete Research

journal homepage: www.elsevier.com/locate/cemconres

Graphic statics and their potential for digital design and fabrication with concrete

Joseph Schwartz

Chair of Structural Design, Institute of Technology in Architecture, ETH Zürich, Stefano-Franscini-Platz 1, 8093 Zürich, Switzerland

ARTICLE INFO

Keywords:
Theory of plasticity
Lower bound theorem
Graphic statics
Reinforced concrete
Parametric design
Form finding
Digital fabrication

ABSTRACT

The paper describes the influence of the lower bound theorem of theory of plasticity on the design of structures during the last 70 years. Starting with the fundamental theorems developed in the middle of the last century, the influence of rigid plastic (discontinuous) stress fields on the design of reinforced beam, wall, slab and shell structures is worked out. The potential of graphic statics for digital design and fabrication is illustrated by the results of several research projects on form finding and design of reinforced concrete structures carried out during the last decade. It is shown that the design methods allow for simultaneous considerations about structure and architecture. This leads to leaner structure and reduced material consumption. However, while a current limitation of manufacturing objects in these "nonstandard" shapes is presently cost limiting, digital fabrication may change this and open a much greater scope for the use and application of graphic statics in a near future.

1. Introduction

Digital fabrication is capturing a lot of attention in all sectors of manufacturing. The construction sector is also examining how related new manufacturing techniques may change the sector. For concrete which is the most used material world-wide after water, various approaches are being pursued as outlined in other papers of this special issue. From the analysis of some of these authors, it appears that concrete mix designs that are adequate for digital fabrication generally lead to an increase in cement content (De Schutter et al., CCR this issue [20]). Because this has an economical and environmental cost penalty, it can be argued that digital fabrication needs to find easy and clear gains elsewhere in order to become competitive. Specifically as summarized by Flatt and Wangler this will mainly come by saving material through lean designs that allow to reduce the amount of material used ([21] this issue of CCR).

It is the objective of this paper to present the approach of graphic statics, which enable a very effective design of such lean structures and for which digital fabrication presents a great opportunity of reducing production costs. Thus, it is hoped that this paper will stimulate necessary exchanges between structural design engineers and the growing community developing digital fabrication with concrete.

At the chair of Structural Design at ETH Zurich, several research projects have been driven in the last ten years showing the potential of using graphic statics to design and fabricate structures with different materials. Most of the results can be applied on concrete structures, as it can be shown that the lower bound theorem of the theory of plasticity is one possible basis for the development of equilibrium solutions. Stress fields as well as the resultant internal forces (struts and ties) of the stress fields are appropriate to visualize the flow of the internal forces and to detail the structural elements. In addition, they are an excellent starting point for the development of form finding methods.

To illustrate the approach we first present applications to basic structural elements and then move on to more elaborate designs of structures, outlining effective form finding strategies. Finally we illustrate activities we have been involved in relating to digital fabrication with concrete. First we present the "Incidental Space" of the Swiss Pavillion at the 2016 Venice Biennale of Architecture. This structure is used as an ultimate example of complexity for which fundamental questions of manufacturing, assembly and structural integrity had to be addressed. This serves as a basis to present ideas and ongoing projects about handling such issues, something which helps to establish a link to challenges in digital fabrication and the link we aspire to establish with structural design.

2. Theory of plasticity

2.1. Plastic design of beam structures

The mathematically closed theory of elasticity is limited to structures with a perfect elastic material behaviour. While the complexity and the abstractness of the elastic calculation methods are pretending

E-mail address: schwartz@arch.ethz.ch.

https://doi.org/10.1016/j.cemconres.2018.06.015
Received 7 February 2018; Received in revised form 18 June 2018; Accepted 19 June 2018
0008-8846/

Please cite this article as: Schwartz, J., Cement and Concrete Research (2018), https://doi.org/10.1016/j.cemconres.2018.06.015

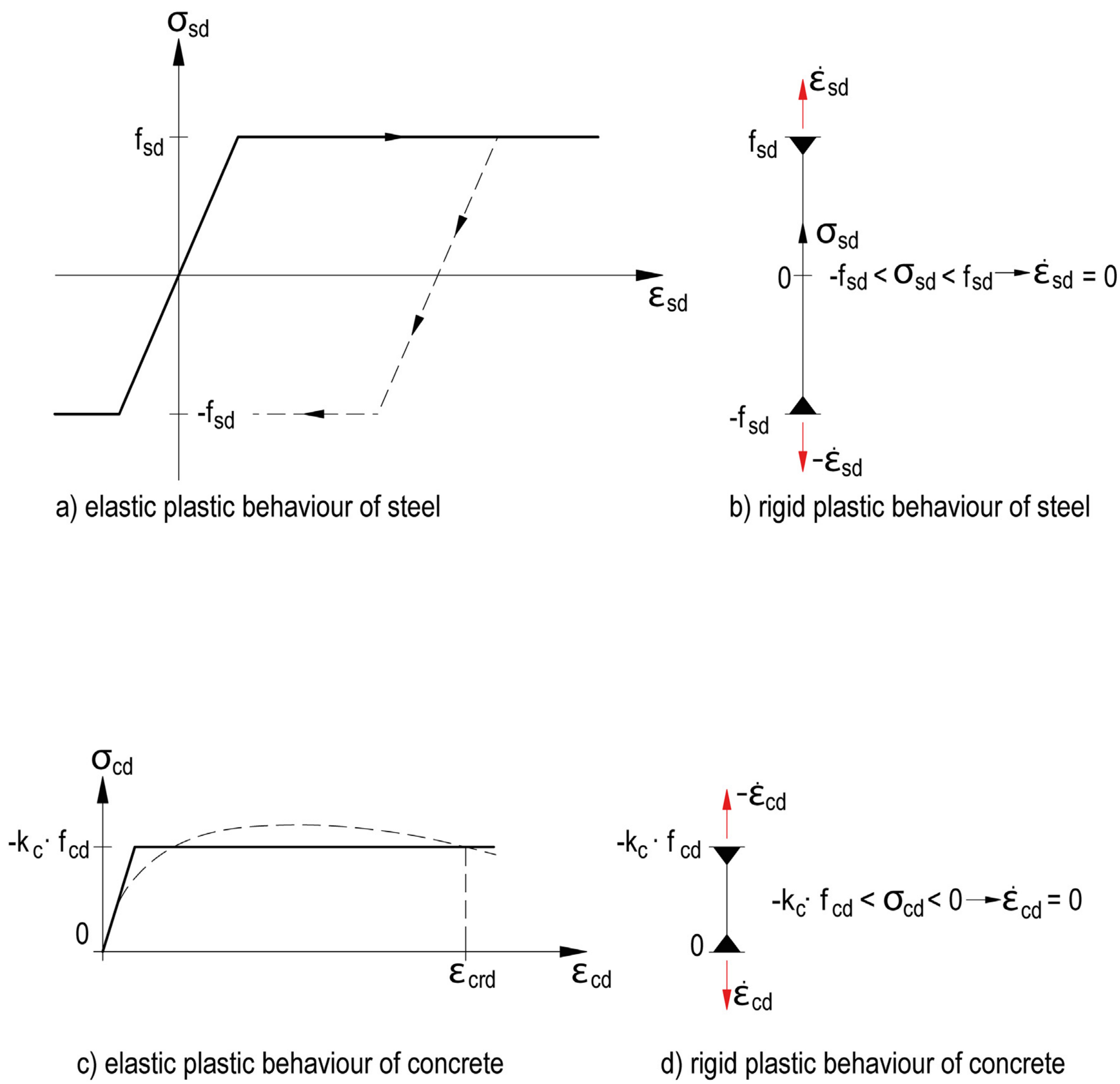

Fig. 1. Idealized stress-strain diagrams for steel and concrete [3].

an exceeding exactness, it cannot be ignored that the results are only correct as long as the material behaviour remains perfectly linear elastic and no residual stresses are occurring. Furthermore, the theory of elasticity is in general not appropriate to determine the ultimate load of structures. Only in the twentieth century, the theory of plasticity has been developed, allowing the determination of the resistance of structures. Prager [1] and Drucker [2] published the Limit State Theorems, and further researchers such as Baker, Heyman, Chen and Thürlimann [3] refined the application of the theory of plasticity on different materials and static problems. Although the limit state theorems of the theory of plasticity have been developed more than fifty years ago, the real meaning of these methods is still poorly understood. The same holds true for the check of the ultimate load of given structures and for the design of new structures with the use of statically admissible stress states. In the last years, further research has been done to define the limit state analysis of 2D statically indeterminate beam, frame and network systems [4] using graphic statics.

2.1.1. *Fundamentals of the theory of plasticity applied on bending systems*

In the theory of plasticity the mechanical behaviour is restricted to those characteristic material values, which for given assumptions essentially determine the strength, i.e. the yielding strength of the reinforcement steel f_{sd} and the effective compressive strength of concrete $k_c \cdot f_{cd}$, where f_{cd} is the cylinder compressive strength and k_c a scaling factor considering the lateral state of strain [3]. If for a particular material the plastic deformations are substantially greater than the elastic deformations, then the latter may be neglected. This assumption is fairly well fulfilled for steel and ductile reinforced concrete structures, stability failure excluded.

Using the corresponding uniaxial stress-strain diagrams for structural steel and concrete (Fig. 1), it is only necessary to specify the strain rate $\dot{\varepsilon}_{sd}$ respectively $\dot{\varepsilon}_{cd}$ i.e. the direction of the strain increment of concrete or steel.

The static behaviour of plane frame and beam structures is characterised above all by the moment-curvature relationship of the section (Fig. 2). The rigid-plastic solution utilizes only the plastic moment M_{Rd} and the plastic curvature increment $\dot{\chi}_d$. If the curvature at the point of reaching the yield moment is small compared with the total curvature at failure, then rigid-plastic behaviour can be assumed.

2.1.2. *The lower bound theorem*

The methods for determining the collapse load using the theory of plasticity are based on the limit state theorems. For the first one, only static considerations are used, i.e. the equilibrium conditions, the static boundary conditions for the beam and the yield condition. Each bending moment distribution M_d (x) which fulfils the equilibrium