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# Orbital angular momentum filter based on multiple-beam interference

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ABSTRACT

By virtue of multiple-beam interference, periodic structures such as photonic or phononic crystals can be used to separate an incident polychromatic light or acoustic beams, respectively, into their constituent frequency components. In addition, it is widely acknowledged that a light beam that is carrying orbital angular momentum (OAM) also has a spatial angular frequency in the azimuthal zone. Here we demonstrate that multiple-beam interference in the azimuthal zone can be used to modulate the transmission of the OAM state and; can thus serve as an OAM filter. The $2D$ transmission spectrum is analyzed and the mode resolution is discussed in detail. We anticipate that this OAM filter will be promising for use in applications such as optical switches, mode de-multiplexing in mode-division multiplexing communications and single-photon discrimination when using linear optical elements.

## 1. Introduction

Light beams with spiral azimuthal phase carry orbital angular momentums (OAM) [1,2] which is characterized by the spiral phase distribution $\exp(il\alpha)$ within the transverse plane, where $l$ is the topological charge which can be any integer, and $\alpha$ is the azimuthal angle. The inherent orthogonality and the arbitrary dimensions of the OAM states have been applied in a variety of areas, including mode-division multiplexing communication [3–8], quantum information systems [9–14] and the production of specific shapes using combinations of multiple beams with complex coefficients [15].

Successful sorting and measurement methods for these OAM states present the first step for real-world applications in most cases and have attracted broad research interests. The first type of method uses the interference within the azimuthal domain. Examples include: the interference of a mode of $exp(il\alpha)$ with its mirror image to produce an interferogram with $2l$ spokes that is used to determine the OAM state [16,17]; modulating of the rotation angles between two arms of a Mach–Zehnder interferometer, which is used to achieve a constructive or destructive interference for different states [18,19]; and analysis of the interference pattern between a light beam and a reference field [20,21]. The second approach uses computer-generated holograms to imprint a helical phase structure on the transverse plane of the light beam [22,23]. Examples include: imprinting a phase pattern with the same OAM number but opposite sign to convert an OAM state front into a plane wave thus eliminating the phase singularity [24]; mapping an azimuthal phase of an OAM mode to a titled planar wavefront using a Cartesian to log-polar transformation [25–27]; use of a hologram with a pattern composed of gratings with gradually changing periods to change a Laguerre–Gaussian beam into a Hermite–Gaussian beam [28]; measurement of the local skew angle of the Poynting vector using a Shack Hartman wave-front sensor [29]; and reshaping of the phase structure of incident light to control scattering processes [30]. In addition, methods based on the OAM-to-polarization coupling effect have also been used to mimic Faraday rotation [31], along with an OAM filter based on use of the polarization degree of freedom [32]. Furthermore, the rotational Doppler effect [33] has been used to analyze OAM mode distribution of an incident light beam.

Use of periodic structures to modulate the propagation properties of waves has previously been studied in depth [34–36], and has stimulated research interest in structures such as photonic [37] and phononic crystals [38]. The introduction of a periodic change into a medium means that the waves that propagate in that medium will have their translation symmetry altered, thus producing the effect of band folding. Essentially, the modulation of transmission properties of waves in periodic structures can be regarded as an interference effect

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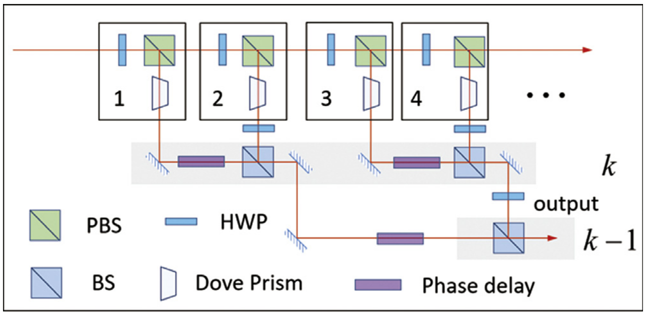


**Fig. 1.** Proposed OAM filter based on multiple-beam interference. For $n$th order multiple-beam interference, a combination of a half-wave plate and a polarizing beam splitter (PBS) is used to divide the incident beam into $n$ equal parts. The Dove prism contained in each branch is used to control the relative azimuthal phase difference $\theta$, while the wave plates are used to compensate for the half-wave loss that is introduced by interface reflection, and a set of phase compensators is inserted to maintain an equal path lengths for these branches. The OAM filter has a reverse-cascade structure, in which every pair of adjacent branches is combined to form a series of MZ interference structures, which are labeled as the $k$th order, and their outputs are then sent to form the next series of MZ interference structures, which are labeled as the $(k-1)$th order. Finally, only two branches are left to form the last MZ interference structure, which is labeled 1. Therefore, the number of branches $n$ satisfies the requirement that $n = 2^k$, where $k$ is a positive integer.

that is caused by multiple reflections at the interface. Theoretically, transmission modulation via multiple interference effects can also be performed on the azimuthal phase of an OAM state.

In this paper, we propose a new approach to filtering of OAM modes based on coherent multiple-beam interference within the azimuthal region. For a light beam that contains a set of OAM modes; that is divided into $n$ parts with equal weight and an evenly distributed azimuthal phase difference $\theta$, the transmission of a coherent combination of these parts shows a strong dependence on $\theta$. At a $\theta$ value of $2\pi/l$, only one kind of modes can attain 100% in transmission, while the other modes are suppressed to transmission of $(2/3\pi)^2$ or less. Based on this principle, an OAM filter with an ideal filtering efficiency of 100% could be realized; in such a filter the number should be set to $2^k$. A full analysis of the $2D$ transmission spectrum and a detailed discussion of the mode resolution are presented in this paper.

## 2. Main text

The OAM mode that we discuss in this paper is the well-studied Laguerre–Gaussian mode (LG) mode, coherent combination of this can form different structured optical vortexes. An LG mode has the following form [39,29]:

$$u_{l,p}(r,\alpha) = \sqrt{\frac{2p!}{\pi(p+|l|)!}}\frac{1}{w(z)}\left(\frac{\sqrt{2}r}{w(z)}\right)^{|l|} L_p^l\left(\frac{2r^2}{w(z)^2}\right)\exp\left(\frac{-r^2}{w(z)^2}\right)$$
$$\exp(il\alpha)\exp\left[i(2p+|l|+1)\arctan\left(\frac{z}{z_R}\right)\right], \tag{1}$$

here $w(z)$ is the beam waist of the fundamental mode, $r$ is the radial coordinate, $\alpha$ is the azimuthal angle, $L_p^l\left(2r^2/w(z)^2\right)$ is the associated Laguerre polynomial, $p$ is the number of radial nodes, $l$ denotes the winding number of the azimuthal phase terms, and $(2p+|l|+1)\arctan\left(z/z_R\right)$ represents the Gouy phase. For ease of discussion, $w(z) = 1$ is set in the theoretical simulation. In real LG beam applications, it is not convenient to manipulate the radial terms, so we have simplified our discussion by setting $p$ to 0 in this work. Because $l$ and $p$ present two independent degree of freedoms of the LG beam, the conclusion that is obtained for the $p = 0$ beams can also be generalized for use with the beams with nonzero $p$ values.

A schematic overview of the proposed setup is shown in Fig. 1. In contrast to the cascade form [18], the OAM filter in this paper has a funnel structure. An incident beam is divided into $n$ branches with equal weight using a set of assemblies composed of a half wave plate and a polarizing beam splitter. These branches are indexed using numbers from 1 to $n$, and an azimuthal angle change of $j\theta$ is brought into the $j$th branch using a Dove prism, this angle change corresponds to a phase variation of $\exp(ijl\theta)$ for the OAM mode $l$. Pairs of adjacent branches are used to form a series of Mach–Zehnder (MZ) interference structures that are labeled as being $k$th order, and their outputs are then sent to form the next series of MZ interference structures, which are labeled as the $(k-1)$th order. The half-wave loss that is caused by interface reflections in each branch is compensated using the half-wave plate. A set of phase delays is inserted to compensate for the possible optical path differences between the different branches. The light beam from the output port is then analyzed and measured. To obtain filter efficiency of 100%, the number of branches of the interference order should satisfy $n = 2^k$. In Fig. 1, for example, four branches take part in the interference process when $k = 2$.

Consider an incident light beam carrying an OAM of $l$; the complex electric field of this beam writes

$$\Psi_l(r,\alpha) = A_l u_{l,0}(r,\alpha), \tag{2}$$

here $A_l$ is the amplitude, and the field at the output port is given as

$$\begin{aligned}\Psi_l(r,\alpha : n,\theta) &= \left(\frac{1}{\sqrt{2}}\right)^k \sum_{m=0}^{n-1}\frac{1}{\sqrt{n}}A_l u_{l,0}(r,\alpha)\exp(iml\theta)\\ &= \frac{1}{\sqrt{n\cdot 2^k}}\frac{1-\exp(inl\theta)}{1-\exp(il\theta)}A_l u_{l,0}(r,\alpha).\end{aligned} \tag{3}$$

Here, $n$ is the number of components into which the incident beam is divided, and $\theta$ is the relative rotated azimuthal angle difference between every pair of adjacent branches. Because the propagation lengths between the input port and the output port are the same for all branches, the influence of the Gouy phase caused by the finite transverse structure of the light beam can be neglected. When $n = 2^k$, we can then write the transmission of the OAM filter as

$$T_l(n,\theta) = \tau_l^*(n,\theta)\,\tau_l(n,\theta) = \frac{1}{n^2}\frac{1-\cos(nl\theta)}{1-\cos(l\theta)}. \tag{4}$$

Here, $\tau_l(n,\theta) = (1-\exp(inl\theta))/n(1-\exp(il\theta))$ is the complex amplitude of the transmission. From Eq. (4) we see that $T_l(n,\theta)$ is a discrete form of a sinc-like function with respect to the variable $l\theta$. When $l$ is fixed, $n$ and $\theta$ can be regarded as a pair of variables that are connected via a Fourier transformation, in a manner similar to the relationships between the time and frequency of a time signal, the momentum and position of a wave undergoing diffraction, or the angular position and orbital angular momentum used for coherent superposition of OAM beams.

The dependence of the transmission $T_l(n,\theta)$ on $n$ is illustrated in Fig. 2. Here, $l = 5$ is chosen to demonstrate this tendency. The transmission function $T_l(n,\theta)$ shows peaks at $2\pi/l$ and its integer multiples. At $n = 2$ (shown in Fig. 2(a)), $T_l(2,\theta)$ has a finite value over a relative broad angular range. However, when $n$ increases to 16, $T_l(16,\theta)$ is then confined within a relative narrow angular range. Here we define the width $W(n,l)$ of $T_l(n,\theta)$ as being the distance between the maximum value and the first node. From Eq. (4), we find that

$$W(n,l) = \frac{2\pi}{nl}. \tag{5}$$

The width of the transmission function is inversely proportional to $m$, as shown in Fig. 2(c). It can be predicted from this that as $n$ increases towards infinity, $T_l(n,\theta)$ will ultimately become a delta-function.

Now we consider the complex light field $\Psi(r,\alpha)$, which is a coherent superposition of the different LG modes

$$\Psi(r,\alpha) = \sum_l A_l u_{l,0}(r,\alpha), \tag{6}$$