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# Dynamic absolute distance measurement by frequency sweeping interferometry based Doppler beat frequency tracking model

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ABSTRACT

We propose a frequency-sweeping interferometry (FSI) with single external cavity diode laser that can be used to measure the absolute distances of a dynamic target. Generally, the FSI-Doppler effect induced by target drift restricts the measurement accuracy severely, but the Doppler effect can also be used to measure the actual movement during optical frequency sweeps. Thus, a mathematical model of the Doppler effect in FSI is presented and a novel FSI method based on Doppler beat frequency tracking model is proposed to realize the absolute distance measurement of a dynamic target. The method was verified successfully by measuring vibrating targets of 50∼5000 Hz at 38 mm in the laboratory.

## 1. Introduction

The future of the industrial production is the combination of the high-resolution measurement and positioning in the process. However, in the case of large volume parts, especially in the aerospace industry, the dimension of the workpiece makes it impractical to work with extra mechanical guidance for the measurement system. The absolute distance measurement (ADM) system can be used for distance measurement of up to several tens of meters and the relative uncertainty is less than $1 \times 10^{-6}$, as opposed to the more common displacement measurement systems [1–25]. A direct interferometric measurement of the actual absolute distance can be realized by an alternative approach, the so-called variable synthetic wavelength or frequency-sweeping interferometry (FSI), which has an unlimited unambiguity range in principle. It is a promising method of absolute distance measurement due to its high-precision and ease of implementation. However typical FSI method suffers from the limitation of a moving target. The FSI model is based on the hypothesis that the measured length keeps constant during the laser sweep. Nevertheless, the FSI method is highly sensitive to variations in the length, and even the smallest mechanical instabilities due to, e.g. drifts or vibrations of the reflector during the laser sweeps, will be amplified in the measurement results and the amplification factor $\gamma$ is the ratio of the synthetic wavelength $\Lambda$ over the swept laser wavelength $\lambda$ ($\gamma = \Lambda/\lambda \approx 2564$ for the parameters in this study) [26–28]. It can be explained by the Doppler effect. Caused by the Doppler effect in FSI, a movement of the target in one optical wavelength is interpreted as a drift in one synthetic wavelength. That means, the FSI method is not able to measure a moving target. Although it is well known that the Doppler effect has been widely applied in the velocity detection of the dynamic target. Nevertheless, in FSI system, the Doppler effect coupling the optical frequency modulation with the target motion cannot achieve the target velocity measurement. To address the Doppler effect problem in FSI, various solutions have been proposed. In 2001, Richard Schneider et al. [16] proposed the dual-FSI system with two simultaneous frequency swept lasers operating in opposite direction, and the amplified drift errors are eliminated by averaging the two phase shifts produced by these two swept lasers. With a single tuned laser source and a frequency-stabilized He–Ne laser, Martinez et al. [12] used four-wave mixing (FWM) as a means to create a second swept light source suitable for dual FSI systems. The generated light is mirror copy of the original swept laser, and the two light beams are synchronized and tuned in opposite directions. The above FSI systems can directly eliminate any drift amplification errors $\Delta l \times \gamma$. However, the actual drift process cannot be measured. To monitor the drifts process and correct the measurement, Warden et al. [9,29] presented a dual-FSI system with gas absorption cell to achieve the dynamic OPD measurement at any sampling point. Pollinger and Liu et al. [13,28] uses an additional heterodyne interferometry to directly measure the target movement and calculate the distance $L$ by combining the FSI measurement. Without utilizing auxiliary laser, based on the hypothesis that the target is drifting at a constant speed or acceleration during the laser sweeps, scientists proposed various algorithms to reduce the Doppler effect by performing consecutive measurements in increasing

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and decreasing laser sweep durations [26,30]. Liu et al. [31,32] established a motion state estimation model of the target mirror based on the FSI measurement model and successfully adopted a Kalman filter to reduce the amplified drift errors. In this model, the measurement rate depended on the modulation period of the ECDL. Due to the physical limitations of the ECDL, it was difficult to realize a high modulation frequency, which drastically limited the measurement rate. In fact, all the compensation algorithms will lose the measurement of the drift process during one laser sweep.

In this paper, we herein propose a new perspective to dynamic absolute distance measurement in FSI system. A mathematical model of the Doppler-FSI is presented, and a single laser FSI system based on Doppler beat frequency tracking model is proposed to realize the dynamic absolute distance measurement in one laser sweep. Its principle and experimental results are shown in the following sections.

## 2. Principle

### 2.1. Basic principle of frequency sweeping interferometry

Fig. 1(a) shows a schematic of the FSI system. The output light of the external cavity diode laser is divided into two parts by a splitter. One part is transmitted into the F-P cavity, which measures the optical frequency sweeping range, while another part is transmitted into the Michelson interferometer. In the interferometer, the beam is split by a beam splitter (BS) and sent along the two arms of the interferometer. One beam is directed along a reference path $D_R$ of fixed length, and the other is directed along the measurement path $D_M$. The measured length $L$ is the half of the optical path difference between the measurement and reference paths. The swept light in the Michelson interferometer requires differing amounts of time to traverse the two arms, and the time difference of which is denoted as $\tau = 2L/c$. As the laser frequency sweeps, the two beams reaching the detector from each arm will have different frequencies, which can be expressed as $\upsilon(t)$ and $\upsilon(t-\tau)$, respectively. The corresponding beams phases can be written as $\phi(t)$ and $\phi(t-\tau)$. And the photocurrent $i(t)$ generated by the photodiode is given by:

$$i(t) \propto \left(P_M P_R \eta\right)^{1/2} \cos(\Phi(t)) \tag{1}$$

where $P_M$ and $P_R$ denote the power of target and reference beams, $\eta$ is the heterodyne efficiency and the beat signal phase can be expressed as $\Phi(t) = \phi(t) - \phi(t-\tau)$.

With the laser frequency sweeping, the $L$ can be determined by the range of varied optical frequency $\Delta\upsilon$ and the corresponding phase difference of the beat signal mixed two beams, as $L = \Delta\Phi \times c/4\pi\Delta\upsilon$. If the distance to be measured is stationary during one measurement, the fixed $L$ can be calculated exactly by counting the numbers $N = \Delta\Phi/2\pi$ of the synthetic wavelength $\Lambda = c/\Delta\upsilon$.

### 2.2. The Doppler effect in FSI

The measurement model introduced above assumes that the measured $L$ is constant. In fact, invariant $L$ is impossible in many measurement environments. For a drifting target, the flight time difference $\tau$ will change with the varying length $L(t)$ and can be expressed as a function of time $\tau(t)$. The term $\tau(t)$ can be rewritten as $2L(t)/c$ and $L(t)$ is the absolute length between the photodetector and the target, which can be expressed as a function of time:

$$L(t) = L_0 + \int^t s\left(t'\right) \mathrm{d}t'. \tag{2}$$

where the term $s(t)$ denotes the speed of the moving target and $L_0$ is the initial distance.

As depicted in Fig. 1(b), for a moving target, the laser frequency $\upsilon(t)$ and $\tau(t)$ change simultaneously. Due to the speed of the target $s(t)$, Doppler effect occurs in the beat signal, and $2s(t)/c$ is the normalized Doppler shift, which can be denoted as $\tau'(t)$.

During measurement, the frequency of the laser diode is modulated linearly over time. Taking into account the nonlinear tuning characteristic of the available tunable laser diodes, the output laser frequency can be expressed as $\upsilon(t) = \upsilon_0 + \beta t + \upsilon_e(t)$, where $\beta$ is the optical frequency tuning rate and the $\upsilon_e(t)$ is the nonlinear term of the laser frequency modulation [33,34].

For the dynamic FSI, the changing phase $\Phi(t)$ of the beat signal can be expressed as:

$$\begin{aligned}\Phi(t) &= \int_{t-\tau(t)}^{t} 2\pi\upsilon\left(t'\right) \mathrm{d}t' \\ &= \int_{t-\tau(t)}^{t} 2\pi\left(\upsilon_0 + \beta t' + \upsilon_e\left(t'\right)\right) \mathrm{d}t' \\ &\approx 2\pi\left(\upsilon_0 + \beta t\right)\tau(t) + \int_{t-\tau(t)}^{t} 2\pi\upsilon_e\left(t'\right) \mathrm{d}t'\end{aligned} \tag{3}$$

Fig. 1(c) shows the changing instantaneous optical phases of the two beams arriving at the photodetector, which depend on not only the swept optical frequency $\upsilon(t)$ of the ECDL but also the varying $L(t)$.

In this case, the instantaneous frequency of the beat signal is given by:

$$\begin{aligned}f(t) &= \frac{1}{2\pi}\frac{\mathrm{d}(\Phi(t))}{\mathrm{d}t} \\ &= \left(\upsilon_0 + \beta t\right)\tau'(t) + \beta\tau(t) + \upsilon_e(t) - \upsilon_e(t-\tau(t))\left(1-\tau'(t)\right)\end{aligned} \tag{4}$$

Next we Taylor expand the function $\upsilon_e(t-\tau(t))$ about $t$:

$$\upsilon_e(t-\tau(t)) = \sum_{n=0}^{\infty} \frac{\upsilon_e^{(n)}(t)}{n!}(-\tau(t))^n \tag{5}$$

If we neglect the second- and higher order terms of the sum in Eq. (5), and this approximation is valid when $\tau(t)^2\upsilon_e''(t) \ll 1$. Then combining Eq. (4) with Eq. (5), the instantaneous frequency can be expressed as:

$$\begin{aligned}f(t) &= \frac{1}{2\pi}\frac{\mathrm{d}(\Phi(t))}{\mathrm{d}t} \\ &= \left(\upsilon_0 + \beta t + \upsilon_e(t)\right)\tau'(t) + (\beta + \upsilon_e'(t))\tau(t) \\ &= \upsilon(t)\tau'(t) + \frac{\mathrm{d}(\upsilon(t))}{\mathrm{d}t}\tau(t)\end{aligned} \tag{6}$$

By combining Eqs. (2) and (6), the mathematical model for the instantaneous frequency $f(t)$ can be rewritten as:

$$f(t) = \frac{2n_{air}}{c}\left(\upsilon(t)s(t) + \upsilon'(t)L(t)\right) \tag{7}$$

Where $\upsilon(t)$ and $\upsilon'(t)$ are the instantaneous optical frequency and the instantaneous optical frequency tuning rate. From Eq. (7), it is clear that the beat frequency of the AD to be measured is the coupling between the distance and the target speed, which causes great interference to the traditional FSI measurement.

### 2.3. The beat tracking model of the FSI with Doppler effect

If the instantaneous frequency of the beat signal caused by absolute distance and the target speed can be separated, the amplified drift errors caused by Doppler can be eliminated and the dynamic measurement can be realized. Thus, the dynamic-FSI measurement can be considered as an estimation for these unknown variables. Various of Bayesian estimation algorithms can be utilized to state estimation. In this paper, a Kalman filter is used as an estimator to implement state estimation. The target state can be defined as a state vector $\boldsymbol{x} = [L, s]^T$, here, a state–space model can be constructed to describe the target state at different moments. The vector $\boldsymbol{x}$ at different sampling moments $\boldsymbol{t}_n$ can be a set $\boldsymbol{X} = \left[\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3, \ldots \boldsymbol{x}_n\right]$. Due to the swept optical frequency, the varied movement states exhibit different characteristics in the frequency domain of the beat signal, the corresponding beat frequency can be