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Relativistic collapse of axion stars

Florent Michel [a,*], Ian G. Moss [b]

[a] Centre for Particle Theory, Durham University, South Road, Durham, DH1 3LE, UK
[b] School of Mathematics, Statistics and Physics, Newcastle University, Newcastle Upon Tyne, NE1 7RU, UK

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ABSTRACT

We study the gravitational collapse of axion dark matter in null coordinates, assuming spherical symmetry. Compared with previous studies, we use a simpler numerical scheme which can run, for relevant parameters, in a few minutes or less on a desktop computer. We use it to accurately determine the domains of parameter space in which the axion field forms a black hole, an axion star or a relativistic Bosenova.

1. Introduction

Amongst the possible dark matter candidates, a coherent scalar field with very low mass is an enticing possibility. The idea originated with the QCD axion [1], but the concept has since been extended to a class of axion-like particles (ALP's) with ultra-light masses [2]. In ALP scenarios, the dark matter forms gravitationally bound objects which may form into galaxy cores [3], or for larger masses into axion mini-clusters [4–6]. These objects are often stable only for a particular mass range, leaving the possibility of detectable cosmological signatures from the axion bound structures or from the remnants of their collapse [3,7].

ALP's are characterised by their mass m and decay constant (or symmetry breaking scale) f. Coherent ALP dark matter scenarios envision the dark matter energy density in the form of large-scale coherent axion oscillations of frequency $\sim m$, with density parameter [1,7]

$$\Omega_{\rm ALP} \sim 0.1 \left(\frac{f}{10^{17}\,{\rm GeV}}\right)^2 \left(\frac{m}{10^{-22}\,{\rm eV}}\right)^{1/2}, \tag{1}$$

although this is rather dependent on initial conditions. Spatial gradients in the oscillating axion field induce "quantum" pressure forces which are capable of supporting structures on the Kpc scale for axion masses around $m \sim 10^{-22}$ eV, or galaxy Halo scales for $m \sim 10^{-24}$ eV [2].

We follow the recent trend of referring to stable axion structures as axion stars (though the term Bose star is also frequently used in this context). So far three distinct scenarios of gravitational collapse for APL's have been identified [8,9]: they can settle down quietly to an axion star; they can radiate away energy in bursts of relativistic axions or they can collapse to a black hole. The second outcome is a relativistic analogue of the Bosenova phenomena in cold-atom physics [10]. Like the cold atoms in a Bosenova, the axions have an attractive self-interaction force which can overcome the quantum pressure. We will use the term Bosenova in this paper to refer to the axion collapse and radiation phenomenon.

The fate of an axion clump can be represented on phase diagrams labelled by parameters describing the axion properties and the initial conditions. Recently, Helfer et al. [9] have produced a phase diagram for spherically symmetric collapse with axion decay constant f and the initial mass of the axion clump, and they have speculated that there is a tricritical point joining phase boundaries between the three outcomes. The aim of this paper is to provide convincing numerical evidence for the tricritical point using a particularly amenable form of the field equations, and to determine the parameter values accurately at the phase boundaries.

We use the null-coordinate integration schemes introduced into spherically symmetric gravitational collapse by Goldwirth and Piran [11]. The null techniques are particularly efficient because the coordinate grid flows inwards with the collapsing matter. For example, the null methods can reproduce the universal scaling phenomena in massless scalar collapse [12], which otherwise is only possible with less efficient mesh refinement techniques [13].

Throughout this work, we use units in which the reduced Planck constant $\hbar$ and velocity of light c are equal to unity. The reduced Planck mass $M_p = (8\pi G)^{-1/2}$, where G is Newton's constant.

* Corresponding author.
E-mail addresses: florent.c.michel@durham.ac.uk (F. Michel), ian.moss@newcastle.ac.uk (I.G. Moss).

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2. Model and field equations

We take the generic axion potential V, which is typical of the potentials which represent axion dark matter [14–16]:

$$V(\phi) = m^2 f^2 \left(1 - \cos\left(\frac{\phi}{f}\right)\right). \tag{2}$$

The parameters m and f are related by (1) if the cosmological dark matter density is in the form of coherent axion oscillations, but we will generally take m and f as free parameters. The Lagrangian density of the axion field is

$$\mathcal{L}_\phi = -\frac{g^{\mu\nu}}{2}(\partial_\mu\phi)(\partial_\nu\phi) - V(\phi), \tag{3}$$

where $g_{\mu\nu}$ is the metric.

The focus of this paper is on spherically-symmetric collapse. Following [11,12,17], we use a very efficient integration scheme obtained by introducing the retarded time coordinate u and radial coordinate r, with metric

$$ds^2 = -g(u,r)\bar{g}(u,r)du^2 - 2g(u,r)du\,dr + r^2 d\Omega^2. \tag{4}$$

As usual, $d\Omega^2$ is the metric on the unit sphere, and we suppose that $g, \bar{g}$ are two smooth functions. Without loss of generality, up to a redefinition of u, we can impose boundary conditions at the origin, $\bar{g}(u,0) = 1$. Imposing that there is no conical singularity at $r = 0$ then implies that $g(u,0) = 1$ [11].

We follow the conventions of [11,12] and introduce the notation $\bar{h}$ for the scalar field ϕ. Radial derivatives of $\bar{h}$ are used to define an auxiliary field h. One can show that the Einstein equations are fully equivalent to a system of first order equations:

$$\partial_u h - \frac{\bar{g}}{2}\partial_r h = \frac{h - \bar{h}}{2r}\left[\left(1 - 8\pi G r^2 V(\bar{h})\right)g - \bar{g}\right] - \frac{g}{2} r V'(\bar{h}), \tag{5}$$

$$\partial_r \ln(g) = \frac{4\pi G}{r}\left(h - \bar{h}\right)^2, \tag{6}$$

$$\partial_r (r\bar{g}) = \left(1 - 8\pi G r^2 V(\bar{h})\right) g, \tag{7}$$

$$\partial_r\left(r\bar{h}\right) = h. \tag{8}$$

The first of these equations is a form of the Klein–Gordon equation which can be integrated using the method of characteristics. This is the only true evolution equation in the system, the other three equations are geometrical constraints.

Starting from the initial data surface $u = 0$, we label the ingoing radial null geodesics by a coordinate v. The ingoing null geodesics for the metric Eq. (5) satisfy the characteristic equation for (5),

$$\partial_u r|_v = -\frac{\bar{g}}{2}. \tag{9}$$

Changing to null coordinates, so that $h(u,r)$ becomes $h(u,v)$, gives the evolution along the characteristic surfaces of constant v,

$$\partial_u h = \frac{h - \bar{h}}{2r}\left[\left(1 - 8\pi G r^2 V(\bar{h})\right)g - \bar{g}\right] - \frac{g}{2} r V'(\bar{h}). \tag{10}$$

In order to solve these field equations, we have adapted the numerical procedure from Refs. [11,18,12]. Starting from given initial data for $\bar{h}$ and r at $u = 0$, we first compute $h(0,v)$, $g(0,v)$, and $\bar{g}(0,v)$ using (6)–(8). We evolve r and h in the u direction using Equations (9) and (10), discarding points for which r becomes negative. At each step, the constraints (6)–(8) are solved by integrating in the v direction.

Evolution methods based on the 3+1 space and time coordinates solve their constraints on the initial time hypersurface, usually as a boundary value problem, and can be subject to constraint violation at later times. This is not an issue with the null coordinate formalism. The method only requires us to solve the *ordinary* differential equations, (10) and (9), with integrations over v at each time step. As a result, the method is remarkably accurate, fast and reliable.

When a black hole forms, it is possible to follow the evolution up to the null surface $u = u_T$ which contains a marginally trapped surface at $r = r_T$. We define the final black hole mass M_H as the Bondi mass [19,20],

$$M_H = \lim_{u\to u_T}\lim_{v\to\infty}\frac{r}{2G}\left(1 - \frac{\bar{g}}{g}\right), \tag{11}$$

since this is appropriate for null coordinate systems. A Schwarzschild black hole metric with mass M, for example, has $g = 1$, $\bar{g} = 1 - 2GM/r$ and $M_H = M$. At the marginally trapped surface $g \to \infty$, and the computational grid has to be compressed to counter the growth in the right hand side of (10). In practice, the integration is stopped when $\bar{g}/g$ reaches a predetermined value. The Bondi mass is calculated at the final value of u and with v at the extreme edge of the coordinate grid.

Removing the limits from (11) gives a local quantity $M_B(u,v)$ which evolves according to

$$\partial_u M_B = -2\pi r^2\left(\frac{2}{g}\left(\partial_u\bar{h}\right)^2 + \bar{g}V(\bar{h})\right). \tag{12}$$

When g, $\bar{g}$, and V are positive, then $\partial_u M_B \le 0$. We will use $-\partial_u M_B$ as a measure of the energy flux from the collapsing star. Any increase of M along in the ingoing null direction indicates (at least if V remains positive) an artefact from the numerical integration, and the corresponding runs are discarded. We can also use (12) to put bounds on the error in the black hole mass from truncating the integration before the trapped surface at $u = u_T$. This gives better control of the black hole mass than we would have using the mass at the trapped surface, $r_T/2G$, which was used in previous work.

3. Numerical results

We preface the full analysis with some results on the collapse of a massive, real, scalar field without self-interaction. Depending on initial conditions, the system can collapse to a black hole or a stable oscillaton, i.e. an oscillating field configuration that maintains its radial profile [6,5]. The phase diagram for relativistic collapse in terms of mass and radius was obtained semi-analytically in Ref. [21]. The fully relativistic collapse of a massive scalar field was studied in some detail using null coordinates in Ref. [17] and using a 3+1 approach in Ref. [22].

We use the null coordinate approach to plot the phase diagram in terms of the axion mass m, the initial radius R and Bondi mass M_B. The choice of initial density profile is somewhat arbitrary, but we choose to work with a Gaussian profile which has been used previously for Bose stars [23]. The scalar field is oscillatory in time, and when projected on to the light-cone in flat space,

$$\bar{h}_i(r) = \sqrt{\frac{2M}{\pi^{3/2}m^2R^3}}\,e^{-r^2/(2R^2)}\cos(mr). \tag{13}$$

The pre-factor has been chosen so that the mass of the star is M in the non-relativistic limit $Rm \gg 1$. The relationship between the radius and the ingoing null coordinate on the initial surface can be specified freely, but the uniform choice $r = 2v$ will be used for simplicity. Initial conditions on the remaining fields are determined by the constraints (6)–(8), which ensure that we have