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Importance of Darcy–Forchheimer porous medium in 3D convective flow of carbon nanotubes

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ABSTRACT

This article explores Darcy–Forchheimer 3D flow of water-based carbon nanomaterial (CNTs). A bi-directional linear stretchable surface has been used to create the flow. Flow in porous space is represented by Darcy–Forchheimer expression. Heat transfer mechanism is explored through convective heating. Results for single-wall (SWCNTs) and multi-wall (MWCNTs) carbon nanotubes have been presented and compared. The reduction of partial differential system into nonlinear ordinary differential system is made through suitable variables. Optimal homotopic scheme is used for solutions development of governing flow problem. Optimal homotopic solution expressions for velocities and temperature are studied through graphs by considering various estimations of physical variables. Skin friction coefficients and local Nusselt number are analyzed through plots. Our findings show that the skin friction coefficients and local Nusselt number are enhanced for larger values of nanoparticles volume fraction.

1. Introduction

Carbon nanotubes are the least complex synthetic arrangement and nuclear holding setup of graphene sheet structures moved up in a state of chamber. Carbon nanotubes have unique physical, electrical, thermal, chemical and mechanical properties, due to the combination of their small size, cylindrical structure and immense surface area. Depending on the number of graphene layers, carbon nanotubes have been subdivided in two types that is single-walled carbon nanotubes (SWCNTs) and multi-walled carbon nanotubes (MWCNTs). Anomalous thermal conductivity enhancement in oil based nanofluids containing carbon nanotube is illustrated by Choi et al. [1]. Xue [2] proposed a model based on Maxwell theory for transport properties of CNTs based composites. Aqueous suspensions of multi-walled carbon nanotubes is explored by Ding et al. [3]. Few contributions in this direction can be reviewed through refs. [4–14] and various studies therein.

Flow of fluid through porous space accept basic part in various mechanical applications, for instance, warm insurance outlining, water advancements in geothermal supplies, underground spreading of substance misuse, nuclear waste document, grain amassing, redesigned recovery of oil stores, enhanced oil recovery, arrive carbon-dioxide sequestration, squeezed cryogenic microsphere assurance, facilitate contact warm exchangers, coal combustors, nuclear waste storage facilities, and warmth pipe development. For flow under low velocity and weak porosity conditions, Darcy developed a pioneering semi-empirical equation. Nonlinearity appears in semi-empirical equation for high Reynolds number which is due to increasing role of inertial forces. Forchheimer [15] predicted a modified equation namely Darcy–Forchheimer equation by introducing quadratic term in momentum equation. Muskat [16] entitled it as Forchheimer factor. Having above in view, further relevant studies on Darcy–Forchheimer flow can be quoted through refs. [17–32] and several studies therein.

The main point of this endeavor is to investigate Darcy–Forchheimer three dimensional (3D) flow of water based carbon nanomaterials. Flow generated is because of linear stretchable surface. Heat transfer mechanism is explored via convective condition. Single-walled (SWCNT) and multi-walled (MWCNT) carbon nanotubes are considered. Xue model is implemented in mathematical modeling. The governing nonlinear system is solved by optimal homotopic approach [33–40]. Effectiveness of sundry variables on velocities and temperature fields are analyzed. Further skin frictions and local Nusselt number are presented via plots.

2. Formulation

We consider three dimension (3D) flow of water based carbon nanomaterials (CNTs) caused by linear stretchable surface in a Darcy–Forchheimer porous space. Temperature at surface is controlled via convection which is described by hot fluid at temperature T_f below the surface and heat transfer coefficient h_f. The surface at $z = 0$ possessing the stretching velocities $U_w(x) = ax$

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Nomenclature

$u,\ v,\ w$	velocity components	CNTs	carbon nanotubes
$x,\ y,\ z$	space coordinates	$U_w,\ V_w$	surface stretching velocities
μ_f	fluid dynamic viscosity	ρ_f	fluid density
ν_f	kinematic fluid viscosity	ν_{nf}	kinematic nanofluid viscosity
k_f	basefluid thermal conductivity	k_{nf}	nanofluids thermal conductivity
α_f	thermal diffusivity of base fluid	α_{nf}	thermal diffusivity of nanofluid
T_f	hot fluid temperature	T_∞	ambient fluid temperature
h_f	heat transfer coefficient	k_{CNT}	CNTs thermal conductivity
$a,\ b$	positive constants	k^*	permeability of porous medium
F	non-uniform inertia coefficient	ϕ	nanomaterial volume fraction
C_b	drag coefficient	$\mathrm{Re}_x,\ \mathrm{Re}_y$	local Reynolds numbers
$C_f,\ C_g$	skin friction coefficients	Nu	local Nusselt number
λ	porosity parameter	η	dimensionless variable
Fr	Forchheimer number	$f',\ g'$	dimensionless velocities
Bi	Biot number	θ	dimensionless temperature
Pr	Prandtl number	α	ratio parameter

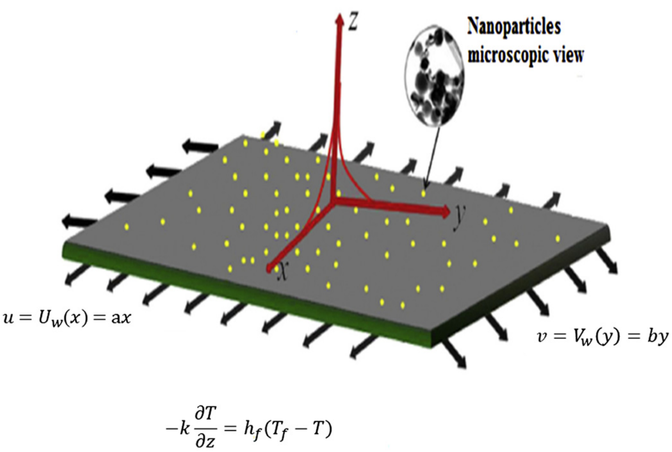

Fig. 1. Physical configuration and coordinate system.

and $V_w(y) = by$ where a and b are the positive constants (see Fig. 1). Single-walled (SWCNTs) and multi-walled (MWCNTs) carbon nanotubes are used as nanoparticles in the water. The problems statements are [11,25]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \nu_{nf}\frac{\partial^2 u}{\partial z^2} - \frac{\nu_{nf}}{k^*}u - Fu^2, \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = \nu_{nf}\frac{\partial^2 v}{\partial z^2} - \frac{\nu_{nf}}{k^*}v - Fv^2, \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \alpha_{nf}\frac{\partial^2 T}{\partial z^2}. \tag{4}$$

The associated boundary conditions are [25]:

$$\left.\begin{array}{l} u = U_w,\ v = V_w,\ w = 0, \\ -k_{nf}\left(\dfrac{\partial T}{\partial z}\right) = h_f(T_f - T) \text{ at } z = 0, \end{array}\right. \tag{5}$$

$$u \to 0,\ v \to 0,\ T \to T_\infty \text{ as } z \to \infty. \tag{6}$$

Note that u, v and w stand for the fluid velocities in the x-, y- and z-directions, ν_{nf} for kinematic viscosity of nanofluid, $F = \frac{C_b}{xk^{*\frac{1}{2}}}$ for nonuniform inertia coefficient of porous space, C_b for drag coefficient, k^* for permeability of porous medium, α_{nf} for thermal diffusivity of nanofluid, T for temperature, T_f for temperature of hot fluid and T_∞ for ambient temperature and c_p for specific heat. Theoretical relation suggested by Xue [2] is defined as follows:

Table 1
Thermophysical aspects of water and CNTs [2].

Physical aspects	Base fluid	Nanoparticles	
	Water	SWCNTs	MWCNTs
ρ	997.1	2600	1600
c_p	4179	425	796
k	0.613	6600	3000

$$\left.\begin{array}{c} \nu_{nf} = \frac{\mu_{nf}}{\rho_{nf}},\ \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \\ \rho_{nf} = (1-\phi)\rho_f + \phi\rho_{CNT},\ \alpha_{nf} = \frac{k_{nf}}{\rho_{nf}(c_p)_{nf}}, \\ \frac{k_{nf}}{k_f} = \frac{(1-\phi)+2\phi\frac{k_{CNT}}{k_{CNT}-k_f}\ln\frac{k_{CNT}+k_f}{2k_f}}{(1-\phi)+2\phi\frac{k_f}{k_{CNT}-k_f}\ln\frac{k_{CNT}+k_f}{2k_f}}, \end{array}\right\} \tag{7}$$

where μ_{nf} stands for viscosity of nanofluid, ϕ for nanoparticle fraction, ρ_{nf} for density of nanofluid, k_{nf} for thermal conductivity of nanofluid, ρ_{CNT} for density of carbon nanotubes and k_{CNT} for thermal conductivity of carbon nanotubes. Table 1 exhibits thermophysical attributes of water and CNTs.

Using the transformations [25]:

$$\left.\begin{array}{c} u = axf'(\eta),\ v = ayg'(\eta),\ w = -(a\nu)^{1/2}(f(\eta)+g(\eta)), \\ \theta(\eta) = \frac{T-T_\infty}{T_f-T_\infty},\ \eta = \left(\frac{a}{\nu}\right)^{1/2}z. \end{array}\right\} \tag{8}$$

Now Eq. (1) is verified while Eqs. (2)–(7) yield

$$\frac{1}{(1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_{CNT}}{\rho_f}\right)}f''' + (f+g)f'' - f'^2 - \frac{\lambda}{(1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_{CNT}}{\rho_f}\right)}f' - Fr(f')^2 = 0, \tag{9}$$

$$\frac{1}{(1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_{CNT}}{\rho_f}\right)}g''' + (f+g)g'' - g'^2 - \frac{\lambda}{(1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_{CNT}}{\rho_f}\right)}g' - Fr(g')^2 = 0, \tag{10}$$

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