Physics Letters A ••• (••••) •••–•••

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Study of partially polarized fractional quantum Hall states

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ARTICLE INFO

Article history:
Received 10 March 2018
Received in revised form 23 May 2018
Accepted 7 August 2018
Available online xxxx
Communicated by R. Wu

Keywords:
Fractional quantum Hall effect
Composite fermion theory
Polarized states of fractional quantum Hall effect

ABSTRACT

We have studied the partially spin polarized fractional quantum Hall states using Chern Simon's theory and plasma picture proposed by Halperin. Using these theoretical techniques we have tried to find the stable polarized states of different filling fractions observed in experiments. We have calculated the ground state energies of those states and also pair correlation function. We have described the nature of the states by the behavior of this quantity. In our study, we have seen that the partially polarized states, which do not fit with Jain's composite fermion description are basically the mixed state of up-spin liquid phase and down-spin solid phase.

The fractional quantum Hall effect (FQHE) [1] is basically the problem of interacting electrons in two-dimensional electron gas (2DEG) in the presence of a strong perpendicular magnetic field. The well established Composite Fermion (CF) theory [2,3] is based on the principle that, in a range of filling factors, each electron in the lowest Landau level (LL) captures an even number of quantum mechanical vortices to the many-particle wave function. The bound state of an electron and vortices behaves the same as a single particle, called the composite fermion [2], which experiences a reduced amount of magnetic field $B^* = B \pm 2p\rho\phi_0$, where $2p$ is an even integer number of flux attachment with each electron, B is an external magnetic field, ρ is the electron (CF) density, and ϕ_0 is the flux quantum. CF's form their own Landau-like kinetic energy levels in this reduced magnetic field, called Λ levels, and their filling factor ν^* is related to the electron filling factor ν through the relation, $\nu = \frac{\nu^*}{2p\nu^* \pm 1}$. In particular, at $\nu = \frac{n}{2pn \pm 1}$, the ground state consists of n filled Λ levels. CF theory explains the FQHE states in details qualitatively. Collective excitation of almost all the filling fraction in the Jain series of positive flux attachment states has been studied earlier [4,5].

In strong magnetic field, spin degree of freedom of electron gets frozen in the direction of the magnetic field. Thus, we obtain fully polarized quantum Hall (QH) states. Partially polarized QH states are found in the experiments [6–9] for relatively small tilted magnetic field, in which the Zeeman splitting energy is small compared to cyclotron energy. Landau level mixing plays an important role in spin polarized FQHS [10], which breaks the particle-hole symmetry. Exact diagonalization [11] method has been used to study the partially polarized states using small number of particles with thermodynamic extrapolation [12]. S. Mandal and Ravishankar proposed a global doublet model [13] and described many body wave function for arbitrary polarized QH states. The most successful theory to explain the partially polarized states of FQHE is the CF theory [14].

In CF picture, the FQHE of filling fraction $\nu = \frac{n}{2pn \pm 1}$ maps into non-interacting n (integer)-number of filled CF Λ-levels, out of that $n_\uparrow$ ($n_\downarrow$) be the number of occupied spin-up (spin-down) CF Landau bands, then the total number of filled Λ-levels $n = n_\uparrow + n_\downarrow$, so that the measure of polarization of the state will be $\gamma = \frac{n_\uparrow - n_\downarrow}{n_\uparrow + n_\downarrow}$ [14]. In this picture we have some limited number of precisely defined polarized states. In Fig. 1 we have explicitly explained the CF polarized states for the filling fraction $\nu = 2/3$ and $2/5$. Panel (A) of this figure represents the fully polarized state, in which only spin-up Λ levels are occupied, unpolarized state is represented in the panel (B), where one spin-up and one spin-down Λ level filled.

Kukushkin, Klitzing and others [15] measured magnetic field dependencies of the electron spin polarization for various filling fractions ($\nu = 2/3, 3/5, 4/7, 2/5, 3/7, 4/9$). Beside the CF polarized states, they observed some specific polarized states which are not explained by the CF picture. The partially polarized states of 2/5 filling fraction has been addressed by Ganpathy Murthy [16] as Hofstadter butterfly problem of charge density wave states of partially filled CF Landau levels (panel (C) of Fig. 1). There is an exact-diagonalization calculation [17,18] for limited number of particles on sphere and torus geometry suggesting anti-ferromagnetic ordered states of $\gamma = 1/2$ at 2/5 and 2/3 filling fractions, also they

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https://doi.org/10.1016/j.physleta.2018.08.008
0375-9601/

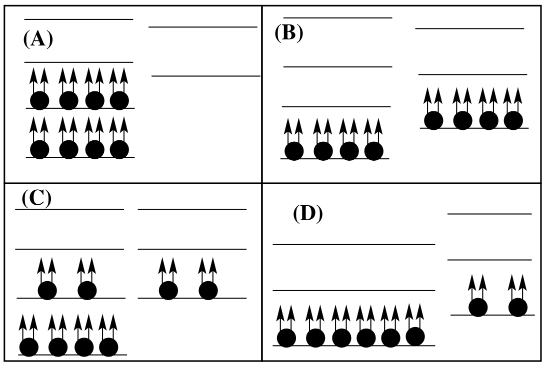

Fig. 1. In each block left panel represents up-spin Λ-level, and right panel represents the down-spin Λ-level. Solid dot with arrow lines represents a CF, electron (solid dot) with even number of magnetic flux attachment (arrow lines). The FQHE filling fraction of electron $\nu = 2/5$ $(2/3)$ maps into $n = 2$ filled Λ level (CF Landau level). (A) Fully polarized state, two spin-up Λ-levels are filled. (B) Unpolarized state, one spin-up Λ-level filled and one spin-down λ-level filled. (C) Partially polarized Murthy's density wave state one spin-up Λ-level filled and one spin-up Λ-level + one spin-down λ-level half filled. (D) partially polarized state, one spin-up Λ-level filled and one spin-down λ-level filled, but the density of states for the up-spin Λ-level and spin-down λ-level are different in such a way that $\gamma = 1/2$.

have demanded a calculation on the large number of particles to understand the occurrence and incompressibility of states. The multi-flavored CF picture, Λ levels within Λ level, has been applied on the partially polarized Hall states outside Jain series [19], though we do not have any clear understanding of attaching different number of flux quanta with the same kind of electrons. In this article we have tried to explain partially polarized states using Chern–Simon's (CS) theory and Plasma (PL) picture.

1. Composite Fermion–Chern Simon's theory

CF is a topological quantity since, they attach quantized vortices with them. The vortices bound to them produce Berry phases, that partly cancel the Aharonov–Bohm phase due to the external magnetic field. By a unitary transformation, so called CS transformation [20] to the electron field operators leads to a topological vector field $\overrightarrow{a}_\alpha$, here α represent spin indices ($\uparrow$ *and* $\downarrow$). The corresponding CS field is given by

$$b_\alpha = 1/e\, \overrightarrow{\nabla} \times \overrightarrow{a}_\alpha = \phi_0 \;\; K_{\alpha\beta} \;\; \rho_\beta \tag{1}$$

here, $K_{\alpha\beta}$ is the two dimensional coupling matrix, ρ_β is the density of electron in the β spin segment, here summation convention has been used. Each species of CS-CF (the quasi-particle under CS transformation) will experience different effective magnetic field. The relation between these mean effective fields and the total applied physical field B is given by

$$B^*_\alpha = B - \phi_0 \;\; K_{\alpha\beta} \;\; \rho_\beta \tag{2}$$

In the magnetic flux attachment picture, we can explain this as an up-spin electron captures K_{11} flux quantum of magnetic field, whereas a down-spin electron captures K_{22} number of flux quantum of magnetic field, and an up-spin electron feels that a down-spin electron is attached with K_{12} number of flux quantum of magnetic field. So K_{11} and K_{22} must be even integers, otherwise we shall lose the Fermionic nature. A special case if we set all the elements equal i.e. $K_{11} = K_{22} = K_{12} = K_{21}$, the state becomes Jain's CF state.

The spin-dependent effective magnetic field creates different set of effective Landau levels with different degeneracy in different spin segment (panel (D) of Fig. 1) as degeneracy is proportional to the magnetic field. Denoting $n_\uparrow$, $n_\downarrow$ as the number of completely filled effective Landau levels (Λ levels) by the spin-up and spin-down species of CS-CFs, one obtains the relation from equation (2),

$$\frac{\rho_\alpha}{n_\alpha} = \frac{\rho}{\nu} - K_{\alpha\beta} \;\; \rho_\beta \tag{3}$$

The polarization is $\gamma = \frac{(\rho_\uparrow - \rho_\downarrow)}{\rho}$ and total density is $\rho = \rho_\uparrow + \rho_\downarrow$.

We have studied Chern–Simon's wave function (k_1, k_2, n) for the SU(2) case, which is described by the exponent matrix [21]

$$K_{SU(2)} = \begin{bmatrix} 2k_1 & n \\ n & 2k_2 \end{bmatrix}$$

where k's and n's are positive integers.

Eliminating all ρ's from equation (3), we find out relation between k_1, k_2, n with polarization and total filling factor ν given below,

$$\frac{1+\gamma}{n_\uparrow} = \frac{2}{\nu} - 2k_1(1+\gamma) - n(1-\gamma) \tag{4}$$

$$\frac{1-\gamma}{n_\downarrow} = \frac{2}{\nu} - 2k_2(1-\gamma) - n(1+\gamma) \tag{5}$$

The different combination of the parameters (k_1, k_2, n) and number of filled Λ levels ($n_\uparrow$ and $n_\downarrow$) give us different polarized states of a particular filling fraction ν.

Wave function: We have considered N number of electrons moving on a spherical surface [22,23] in presence of radial magnetic field of total flux $2Q\phi_0$ created by a Dirac monopole at the center of sphere with monopole strength Q. The radius of the sphere is $R = \sqrt{Q}$, in units of magnetic length $l = \sqrt{\hbar c/eB}$. The effective magnetic flux experience by the composite particles can be expressed from equation (2) as

$$2q_\alpha = 2Q - \sum_\beta (N_\beta - \delta_{\alpha\beta}) K_{\alpha\beta} \tag{6}$$

$$\begin{aligned} \Rightarrow Q &= \frac{1}{4}\left(\sum 2q_\alpha + \sum 2k_\alpha(N_\alpha - 1) + nN\right) \\ &= \frac{1}{4}(2q_1 + 2q_2 + 2k_1(N_1 - 1) + 2k_2(N_2 - 1) + nN) \end{aligned}$$

The above relation is very important to set the radius of the spherical surface. Here, N_1 (N_2) is the number of spin up (down) electrons. The variational wave function for the state is proposed by S. Mandal and coworker [21] and is given by

$$\begin{aligned} \Psi_{k_1,k_2,n} = {} & \mathcal{P}_L \Phi_{n_\uparrow}(\Omega^{(1)}_1, \cdots, \Omega^{(1)}_{N_1}) \Phi_{n_\downarrow}(\Omega^{(2)}_1, \cdots, \Omega^{(2)}_{N_2}) \\ & \times J_{11}\; J_{22}\; J_{12} \end{aligned}$$

where $\Phi_{n_\uparrow}$ is the Slater determinant of $n_\uparrow$ filled CFs Λ level, $\Omega^{(i)}_1, \cdots, \Omega^{(i)}_N$ are the positions of CFs on the spherical surface, upper index indicate the different species of CFs and the Jastrow factor is given by

$$J_{12} = \prod_{i,j}^{N_1,N_2} (u^{(1)}_i v^{(2)}_j - u^{(2)}_j v^{(1)}_i)^n$$

$$J_{\alpha\alpha} = \prod_{i<j}^{N_\alpha} (u^{(\alpha)}_i v^{(\alpha)}_j - u^{(\alpha)}_j v^{(\alpha)}_i)^{2k_\alpha}$$

where the spinor variables are $u_i = cos(\theta_i/2)exp(-i\phi_i/2)$ and $v_i = sin(\theta_i/2)exp(i\phi_i/2)$ with $0 \le \theta_i \le \pi$ and $0 \le \phi_i \le 2\pi$. $\mathcal{P}_L$ is the lowest Landau level projection operator [5,24].

Please cite this article in press as: M. Indra et al., Study of partially polarized fractional quantum Hall states, Phys. Lett. A (2018), https://doi.org/10.1016/j.physleta.2018.08.008