Ocean Modelling 130 (2018) 40–47

Contents lists available at ScienceDirect

Ocean Modelling

journal homepage: www.elsevier.com/locate/ocemod

The dependence of energy dissipation on spatial resolution in a viscous-plastic sea-ice model

James Williams*[,a,b], L. Bruno Tremblay[c]

[a] Department of Earth, Atmospheric and Planetary Sciences, Massachusetts Institute of Technology, 54-918, 77 Massachusetts Avenue, Cambridge, MA 02139-4307, USA
[b] NASA Goddard Institute for Space Studies, 2880 Broadway, New York, NY 10025, USA
[c] Department of Atmospheric and Oceanic Sciences, McGill University, 805 Sherbrooke Ouest, Montréal, QC, H3A 2K6, Canada

ARTICLE INFO

Keywords:
Sea ice
Arctic
Numerical modeling
Rheology

ABSTRACT

We present sea-ice kinetic energy budgets to quantify the relative importance of the various energy sinks in a viscous-plastic sea-ice model. To this end, we study two idealized model domains where energy dissipation associated with shear and axial (ridge/lead building) deformation can be analyzed independently. We find that when only shear deformation is present - either at the domain boundary induced by the no-slip boundary condition or within the model domain induced by gradients in the surface air stress - the energy dissipated through friction reduces in relative importance as the spatial resolution of the model is increased. In the limit where the spatial resolution tends to zero, the simulated sea ice drift tends to the analytical solution - giving us confidence in the numerical implementation of the governing differential equation. Increasing spatial resolution leads to a localization of deformation along the shear lines effectively increasing the area over which energy is input by the wind which is not compensated for by frictional shear dissipation. For instance at 40 km spatial resolution, 64% and 29% of the input power is dissipated through shear deformation and water drag respectively, while at 5 km spatial resolution 54% and 43% of the input power is dissipated by the respective processes. These values approach the respective values of 53% and 47% found analytically for this particular model configuration. The overall result is a 64% increase in the domain total sea-ice kinetic energy when the spatial resolution is increased from 40 km to 5 km due to the finer representation of shear lines. In convergence, the mean kinetic energy and potential energy do not depend meaningfully on the spatial resolution. In this case, the structure of the thickness and concentration fields effectively sets the velocity gradient near the boundary provided that the plastic deformation wave associated with the ridge building process is resolved.

1. Introduction

The power input by the surface winds into the Arctic sea-ice cover is the main source of its kinetic energy (drift speed). In the widely used Viscous-Plastic (VP) sea-ice model of Hibler (1979), this energy is largely dissipated by water drag during the summer months. During the winter months, kinetic energy dissipation due to friction associated with ridging and shear become important as well (Bouchat and Tremblay, 2014). For instance, Pritchard (1981) found that roughly one third of the power input by the surface wind was lost to the ocean in the Beaufort Sea during winter, while the remaining power was assumed to be dissipated by internal ice friction. Furthermore, Pritchard (1988) showed that the partitioning of shear and divergent deformation in a plastic model is related to the shape of the yield curve and the associated flow rule used to define the sea-ice rheology.

Sea-ice deformation dissipates energy through friction in both shear and ridging. Kinetic energy is also converted to potential energy during ridge building as work must be done to raise the ice against gravity. Bouchat and Tremblay (2014) found that in the VP sea-ice model the majority of the power input is transferred to the ocean with approximately 15% of the Arctic wide annual power input being dissipated by internal ice mechanics. The authors found that, of that 15%, frictional shear dissipation accounts for roughly 75% of the internal sea-ice stress dissipation in March, with the frictional ridging dissipation and potential energy increase accounting for 15% and 10% respectively.

Recently, Spreen et al. (2016) showed that in a coupled ice-ocean model forced by reanalysis data, the magnitude of Arctic averaged sea-ice deformation rates at a variety of spatial resolutions are approximately 50% lower in the VP model when compared to the RADARSAT Geophysical Processor System (RGPS) satellite observations. The

* Corresponding author.
E-mail address: jwilli@MIT.edu (J. Williams).

https://doi.org/10.1016/j.ocemod.2018.08.001
Received 8 June 2017; Received in revised form 11 June 2018; Accepted 2 August 2018
Available online 10 August 2018
1463-5003/

authors found that there was a 6% increase in the areal mean deformation rate when the spatial resolution was increased from 18 km to 4.5 km. This indicates that increasing the spatial resolution leads to better agreement between simulated and observed deformation statistics.

In this paper, we quantify the power input and dissipation in the VP sea-ice model as the spatial resolution is increased. To this end, we study two idealized model domains and forcing fields; one where energy is dissipated only in shear and one where energy is dissipated primarily in convergence/divergence (ridge building or lead opening). In these idealized situations, we can solve the steady-state sea-ice momentum equation analytically. In the case of pure shear, we find that the simulated mean sea-ice velocity is far from the analytical solution at coarse spatial resolution (40 km) but approaches the analytic solution as the spatial resolution is increased. We find that this increase in the sea-ice velocity is due to a reduction in the relative importance of frictional energy dissipation in shear compared to water drag dissipation. When divergence (or convergence) is the primary mode of failure, the numerical solution does not show a strong dependence on spatial resolution.

2. Model description

The 2-d sea-ice momentum balance is given by:

$$\rho_i h\frac{\partial \mathbf{u}}{\partial t} + \rho_i hf\hat{\mathbf{z}}\times\mathbf{u} = \boldsymbol{\tau}_\mathbf{a} - \boldsymbol{\tau}_\mathbf{w} - \rho_i hg\nabla H_d + \nabla\cdot\boldsymbol{\sigma}, \tag{1}$$

where ρ_i is the density of sea ice, h is the grid-cell mean sea-ice thickness, $\mathbf{u}$ is the 2-d horizontal sea-ice velocity vector, f is the Coriolis parameter, $\boldsymbol{\tau}_\mathbf{a}$ is the surface air stress, $\boldsymbol{\tau}_\mathbf{w}$ is the stress imparted on the bottom of the ice by the ocean, g is the gravitational acceleration, H_d is the sea surface dynamic height and σ is the vertically integrated 2-d internal ice stress tensor.

Using quadratic drag laws (e.g. McPhee, 1982), the effective surface air and ocean stress acting on the ice, $\boldsymbol{\tau}_\mathbf{a}$ and $\boldsymbol{\tau}_\mathbf{w}$, are given by:

$$\boldsymbol{\tau}_\mathbf{a} = \rho_a C_{da}|\mathbf{u}_\mathbf{a}^\mathbf{g}|(\mathbf{u}_\mathbf{a}^\mathbf{g}\cos\theta_a + \hat{\mathbf{z}}\times\mathbf{u}_\mathbf{a}^\mathbf{g}\sin\theta_a), \tag{2}$$

$$\boldsymbol{\tau}_\mathbf{w} = \rho_w C_{dw}|\mathbf{u} - \mathbf{u}_\mathbf{w}^\mathbf{g}|[(\mathbf{u} - \mathbf{u}_\mathbf{w}^\mathbf{g})\cos\theta_w + \hat{\mathbf{z}}\times(\mathbf{u} - \mathbf{u}_\mathbf{w}^\mathbf{g})\sin\theta_w], \tag{3}$$

where $\mathbf{u}_\mathbf{a}^\mathbf{g}$ and $\mathbf{u}_\mathbf{w}^\mathbf{g}$ are the geostrophic velocities of the atmosphere and ocean, θ_a and θ_w are the turning angles of the atmosphere and ocean, and $\hat{\mathbf{z}}$ is the unit vector normal to the ice surface. Note that in Eq. (2) we have assumed that the wind speed is much greater than the sea-ice drift speed, i.e. $|\mathbf{u}_\mathbf{a}^\mathbf{g}| >> |\mathbf{u}|$.

Following Hibler (1979), we assume that the sea ice behaves as a viscous plastic (VP) material with an elliptical yield curve and normal flow rule. The internal stress tensor for the VP constitutive law is written as a function of the strain rates ($\dot\epsilon_{ij}$) and the parameterized ice strength, P:

$$\sigma_{ij} = 2\eta\dot\epsilon_{ij} + \left[(\zeta - \eta)\dot\epsilon_{kk} - \frac{P}{2}\right]\delta_{ij}, \tag{4}$$

where the strain rates ($\dot\epsilon_{ij}$) are given by: $\dot\epsilon_{i,j} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$ and δ_{ij} is the Kronecker delta function. In a plastic material, the bulk and shear viscosities, ζ and η, follow from the choice of yield curve and flow rule. For the VP model of Hibler (1979) they are given by:

$$\zeta = \frac{P}{2\Delta}, \tag{5}$$

$$\eta = \frac{\zeta}{e^2} \tag{6}$$

where $\Delta = \sqrt{(\dot\epsilon_{11} + \dot\epsilon_{22})^2 + \left(\frac{\dot\epsilon_{11} - \dot\epsilon_{22}}{e}\right)^2 + \left(\frac{2\dot\epsilon_{12}}{e}\right)^2}$ and e is the aspect ratio of the yield curve. Note that Δ can be written as a function of the strain rate invariants as:

$$\Delta = \sqrt{\dot\epsilon_I{}^2 + e^{-2}\dot\epsilon_{II}^2}, \tag{7}$$

where $\dot\epsilon_I$ is the mean compressive strain rate (i.e. the divergence) and $\dot\epsilon_{II}$ is the maximum shear strain rate. This notation will prove useful.

In the limit where Δ goes to zero, ζ and η tend to infinity. In this case we use the regularization of ζ proposed by Lemieux and Tremblay (2009):

$$\zeta = \zeta_{\max}\tanh\left(\frac{P}{2\Delta\zeta_{\max}}\right), \tag{8}$$

where tanh is the hyperbolic tangent function, and $\zeta_{\max} = 2.5\times 10^8 P$. This regularization allows for the viscous coefficients to vary smoothly with the strain rates as the local stress state transitions from viscous to plastic and vice versa.

The mass and momentum conservation laws are coupled via the ice strength, P:

$$P = P^* h\exp[-C(1 - A)], \tag{9}$$

where P^* is the sea-ice compressive strength and C is the sea-ice concentration dependence parameter (Hibler, 1979). Eq. (9) is first used to compute the viscous coefficients, ζ and η. The replacement pressure method is then used to recalculate the ice strength as $P = P_R = 2\Delta\zeta$ using ζ from Eq. (8). Note that P_R only differs from P as calculated from Eq. (9) when the ice is in the viscous regime - i.e. grid cells where $\zeta = \zeta_{\max}$ (Hibler and Ip, 1995). The recalculated ice strength is then used in the constitutive law (Eq. 4). The replacement pressure method insures that the ice does not deform due to gradients in the sea-ice thickness or strength in the viscous regime of the model. The corollary to this is that the sea-ice thickness can go slowly to infinity when the forcing is relatively weak but persistent (Kimmritz et al., 2017).

The temporal evolution of h and A are governed by two continuity equations which can be written as:

$$\frac{\partial h}{\partial t} + \nabla\cdot(h\mathbf{u}) = S_A, \tag{10}$$

$$\frac{\partial A}{\partial t} + \nabla\cdot(A\mathbf{u}) = S_h, \tag{11}$$

where S_A and S_h are thermodynamic source terms for the sea-ice concentration and thickness respectively. In the following, we consider only dynamic effects ($S_A = S_h = 0$). The values of parameters used in this model are defined in Table 1 unless otherwise noted in the text.

3. Kinetic energy equation

Following Bouchat and Tremblay (2014), the kinetic energy of the VP sea-ice model can be written as:

$$\frac{\partial K}{\partial t} = H + U = \frac{1}{2}\rho(\mathbf{u}\cdot\mathbf{u})\frac{\partial h}{\partial t} + \mathbf{u}\cdot\left(\rho h\frac{\partial \mathbf{u}}{\partial t}\right), \tag{12}$$

where K is the local kinetic energy per unit area ($K = \frac{1}{2}\rho h\mathbf{u}\cdot\mathbf{u}$). The term H is associated with changes in inertia due to increasing or

Table 1
Definition and values of physical constants and sea-ice model parameters.

Symbol	Definition	Typical value
ρ_i	Density of sea ice	900 kg m^{-3}
ρ_a	Density of air	1.3 kg m^{-3}
ρ_w	Density of sea water	1026 kg m^{-3}
C_{da}	Air drag coefficient	1.2×10^{-3}
C_{dw}	Water drag coefficient	5.5×10^{-3}
P*	Ice strength parameter	27.5×10^{3} N m^{-2}
C	Ice concentration parameter	20
e	Ellipticity of the yield curve	2
g	Gravitational acceleration	9.81 ms^{-2}