

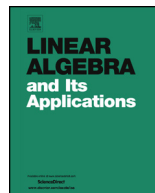


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Engel subnormal subgroups of skew linear groups

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ABSTRACT

Let D be a division ring with uncountable center and n a natural number. In this article we show that every Engel subnormal subgroup of $\text{GL}_n(D)$ is central. This affirmatively answered a question posed in Ramezan-Nassab and Kiani (2012) [8], in the uncountable center case.

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1. Introduction

Let G be a group. For x, y in G , define $[x, {}_1y] = [x, y] = x^{-1}y^{-1}xy$, and inductively, $[x, {}_{k+1}y] = [[x, {}_ky], y]$ for each natural number k . An element $a \in G$ is called *left Engel* if for each $g \in G$, there exists a natural number $n = n(g)$, depending on g , such that

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$[g, {}_n a] = 1$. If $n \in \mathbb{N}$ is such that the relation $[g, {}_n a] = 1$ holds for each $g \in G$, then a is said to be *left n -Engel*. Denote the set of all left Engel and left n -Engel elements of G respectively by $L(G)$ and $L_n(G)$. The group G is called Engel if $G = L(G)$, and called *bounded Engel* if $G = L_n(G)$ for some $n \in \mathbb{N}$.

It is clear that every nilpotent group is n -Engel for some natural number n , and hence every locally nilpotent group is an Engel group; but there exist Engel groups that are not locally nilpotent (see [2]). There are many papers with conditions that force an Engel (n -Engel) group to be locally nilpotent. For example, Gruenberg showed that every solvable Engel group is locally nilpotent [5]. An interesting result due to Suprunenko and Garaščuk in [10] asserts that every linear Engel group is locally nilpotent. But Engel skew linear groups are complicated to deal with; the general backgrounds of Engel skew linear groups can be found in Chapter 3 of [9].

Let D be a division ring and let N be a subnormal subgroup of D^* , the multiplicative group of D . In [6], Huzurbazar showed that if N is locally nilpotent, then it is central, i.e., N is contained in the center of D . In this direction, in [8], authors posed the following question:

Question. Let D be a division ring and let N be a subnormal subgroup of D^* . If N (or D^*) is an Engel group, is it central?

In that paper, authors showed that this question has a positive answer if we assume N to be a bounded Engel group (see [8, Theorem 1.1]), or if we let D be a locally finite-dimensional division algebra over its center (see [8, Theorem 1.3]).

In this paper, we show that this question has an affirmative answer if we additionally assume that the center of D is uncountable:

Theorem 1.1. *Let D be a division ring with uncountable center and N be a subnormal subgroup of D^* . Then $L(N)$ is central.*

Corollary 1.2. *Let D be a division ring with uncountable center and n a natural number. Then every Engel subnormal subgroup of $\text{GL}_n(D)$ is central.*

For bounded Engel elements we have a stronger result:

Proposition 1.3. *Let D be an infinite division ring, n a natural number and N a subnormal subgroup of $\text{GL}_n(D)$. Then $L_m(N)$ is central for each $m \in \mathbb{N}$.*

2. The proofs

In this section we prove our above results. Some ideas come from [3]. Throughout D is a division ring with center F , and D^* is the multiplicative group of D . We begin with simple lemma as:

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