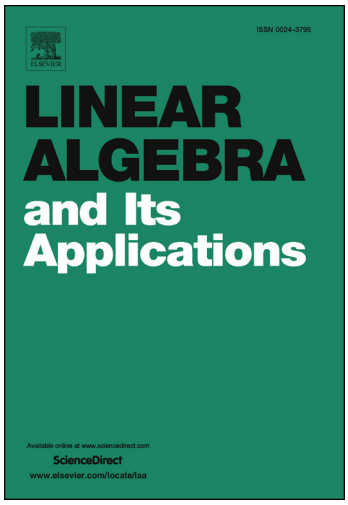

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Inverse numerical range and determinantal representation

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Abstract The numerical range $W(A)$ of an $n \times n$ matrix A is the collection of quadratic forms $z = \xi^* A\xi$ over unit sphere $||\xi|| = 1$. The inverse numerical range problem aims to find a unit vector ξ which corresponds to a given point z of the numerical range $W(A)$. Given a matrix A, the Helton-Vinnikov theorem produces a symmetric matrix S so that A and S have the same numerical range $W(A) = W(S)$. In this paper, we investigate the inverse numerical range problem for the boundary points and points on the boundary generating curve of the numerical range. In place of the construction of unit vectors ξ satisfying $z = \xi^* A\xi$, we express the kernel vector function ξ of the linear pencil $x\Re(S) + y\Im(S) + zI_n$ as a function on the Abel-Jacobi variety of the associated elliptic curve of A. The kernel function plays a key role for the inverse numerical problem. We perform this process when S is a generic 3×3 symmetric matrix.

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Keywords: Inverse numerical range, determinantal representation, elliptic curve, kernel vector function.

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