Journal of Computational and Applied Mathematics 347 (2019) 49–61

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

The risk-neutral stochastic volatility in interest rate models with jump–diffusion processes

L. Gómez-Valle, J. Martínez-Rodríguez *

Departamento de Economía Aplicada e IMUVA, Facultad de Ciencias Económicas y Empresariales, Universidad de Valladolid, Avenida del Valle de Esgueva 6, 47011 Valladolid, Spain

ARTICLE INFO

Article history:
Received 18 January 2018
Received in revised form 27 June 2018

JEL classification:
G13
G17

Keywords:
Interest rate model
Jump–diffusion stochastic processes
Stochastic volatility
Risk-neutral measure
Numerical differentiation
Nonparametric estimation

ABSTRACT

In this paper, we consider a two-factor interest rate model with stochastic volatility and we propose that the interest rate follows a jump–diffusion process. The estimation of the market price of risk is an open question in two-factor jump–diffusion term structure models when a closed-form solution is not known. We prove some results that relate the slope of the yield curves, interest rates and volatility with the functions of the processes under the risk-neutral measure. These relations allow us to estimate all the functions with the bond prices observed in the markets. Moreover, the market prices of risk, which are unobservable, can be easily obtained. Then, we can solve the pricing problem. An application to US Treasury Bill data is illustrated and a comparison with a one-factor model is shown. Finally, the effect of the change of measure on the jump intensity and jump distribution is analysed.

1. Introduction

Understanding the interest rate dynamics and obtaining the term structure of interest rates is important for both practical and theoretical reasons. On the one hand, it is necessary to price and hedge fixed-income derivatives and, on the other hand, it reflects market participant expectations about interest rates changes and their assessment of the monetary policy conditions. Therefore, modelling the term structure of interest rates has been the object of many studies by economists and financial institutions.

Traditionally, the financial literature assumes that interest rates move continuously and they are modelled as diffusion processes, as in [1,2] and so on. However, more recent studies have showed that interest rates contained unexpected discontinuous changes, see for example [3,4]. Jumps in interest rates are, probably, due to different market phenomena such as surprises or shocks in foreign exchange markets. Moreover, when pricing and hedging financial derivatives jump–diffusion models are very important, since ignoring jumps can produce hedging and pricing risk, see [5].

It is widely known that one-factor interest rate models [1–3] are very attractive for practitioners because of its simplicity and computational convenience. However, these models have also unrealistic properties. First, they cannot generate all the yield curve shapes and changes that we can find in the markets. Second, the changes over infinitesimal periods of any two interest-rate dependent prices will be perfectly correlated. Finally, as Hong and Li [6] show, none of their analysed one-factor models adequately captures the interest rate dynamics. Therefore, we consider that at least two factors are necessary to model the term structure of interest rates, such as [7,8]. In fact, the number of factors must be a compromise between numerical efficient implementation and the capability of the model to fit data.

* Corresponding author.
E-mail addresses: lourdes@eco.uva.es (L. Gómez-Valle), julia@eco.uva.es (J. Martínez-Rodríguez).

https://doi.org/10.1016/j.cam.2018.07.048
0377-0427/

In the financial literature, for pricing financial derivatives the state variables must be considered under the risk-neutral measure because of the no-arbitrage arguments. That is, in order to obtain the derivative prices, the market is assumed to be risk-neutral and a change from the physical measure to a risk-neutral measure is necessary. However, the observations in the market are under the physical measure instead of under the risk-neutral measure, therefore, the estimation cannot be done directly from data in the markets. The relation between the processes under the risk-neutral and the physical measure is based on the market prices of risk. If the market prices of risk would be known, then we could easily estimate the functions of the processes under the risk-neutral measure. However, the market prices of risk are not observable either.

In the diffusion literature, this problem is solved by considering portfolios without risk by means of no-arbitrage arguments, see [9] and [10] among others, but their estimation is still complex and provides low accuracy. However, in jump–diffusion models this is not possible because the market is not complete.

Recently, in the literature, a novel approach has been considered for estimating the risk-neutral processes in one-factor term structure models directly from data in the market. In particular, Gómez-Valle and Martínez-Rodríguez [11] proposed a new approach to estimate the risk-neutral functions in a short-rate model but, as usual in the literature, they assumed that jump size distribution did not change under the risk-neutral measure. That is, the market price of risk was assumed to be artificially absorbed by the change of measure of the jump intensity, see also [12]. Johannes [4] takes into account the change of measure in the jump intensity and jump size distribution by means of some arbitrarily chosen parameters. He analyses the dependence of these parameters over the yield curve, but does not provide any estimation procedure. Later [13,14] proposed new results for estimating the functions of the risk-neutral processes for a two-factor commodity futures pricing model. However, it is not possible to apply these results to estimate the functions in a two-factor term structure model. The relations between the financial derivatives and their stochastic variables change depending on the type of the derivative and underlying asset.

Therefore, the main goal of this paper is twofold. First, we obtain some results to estimate the whole risk-neutral functions of a two-factor term structure model directly from data in the market. Then, we show the supremacy of this approach over a short-rate model as well as the importance of assuming different functions and parameters under the risk-neutral measure to price interest rate derivatives.

The rest of the paper is organized as follows. Section 2 develops a two-factor jump–diffusion model with stochastic volatility to price interest rate derivatives. Section 3 provides some results to estimate the whole functions of a pricing model with jumps, directly from market data, even when a closed-form solution is not known. Section 4 shows how to implement the approach in Section 3 with a nonparametric technique. Section 5 shows an empirical application to price zero-coupon bonds and bond options with data from US. Finally, Section 6 concludes. All the implementation has been done using MATLAB software.

2. The jump–diffusion model

In this section, we introduce the two-factor jump–diffusion model that we use to price interest rate derivatives. This research assumes that the state variables are the dynamics of the instantaneous interest rate, r, and the volatility, V.

Define $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ as a complete filtered probability space which satisfies the usual conditions where $\{\mathcal{F}_t\}_{t\geq 0}$ is a filtration, see [15–17].

In order to take into account the abrupt changes of the interest rates in the markets, we consider that the instantaneous interest rate follows a jump–diffusion stochastic process and the volatility, a diffusion process. Therefore, we consider that the factors of the model follow this joint stochastic process[1]:

$$r(t) = r(0) + \int_0^t \mu_r(r(z), V(z))dz + \int_0^t V(z)dW_r(z) + \int_0^t c(r(z-), V(z))dJ(z), \tag{1}$$

$$V(t) = V(0) + \int_0^t \mu_V(r(z), V(z))dz + \int_0^t \sigma_V(r(z), V(z))dW_V(z), \tag{2}$$

where μ_r and μ_V are the drifts and σ_V the volatility of the implied volatility process. Moreover, W_r and W_V are Wiener processes and the impact of the jump is given by the function c and the compound Poisson process, $J(t) = \sum_{i=1}^{N(t)} Y_i$, with jump times $(\tau_i)_{i\geq 1}$, where the jump sizes $Y_1, Y_2, \ldots$ form a sequence of identically distributed random variables with a Normal probability distribution Π, $\mathcal{N}(0, \sigma_Y)$ and $N(t)$ represents a Poisson process with intensity $\lambda(r, V)$. We assume that W_r, W_V and the jump size distribution are independent of N, but the standard Brownian motions are correlated with

$$[W_r, W_V](t) = \rho t.$$

We also assume that the jump magnitude and jump arrival times are uncorrelated with the diffusion parts of the processes. Lastly, we suppose that the functions μ_r, μ_V, σ_V, λ and Π satisfy suitable regularity conditions provided in the Appendix.

[1] r is right-continuous (cadlag, see [16]) and we denote the left limit $r(t-) = \lim_{z\uparrow t} r(z)$. However, for notational clarity the pre-jump values $r(t-)$ will be added only when necessary to avoid confusion and otherwise, they will be assumed implied.