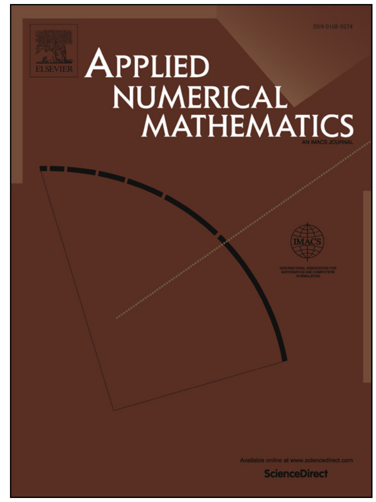
Accepted Manuscript



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PII: S0168-9274(18)30149-1
DOI: https://doi.org/10.1016/j.apnum.2018.07.001
Reference: APNUM 3392

To appear in: *Applied Numerical Mathematics*

Received date: 5 January 2017
Revised date: 11 June 2018
Accepted date: 2 July 2018

A novel finite difference discrete scheme for the time fractional diffusion-wave equation

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Abstract

In this article, we consider initial and boundary value problems for the diffusion-wave equation involving a Caputo fractional derivative(of order α, with $1 < \alpha < 2$) in time. A novel finite difference discrete scheme is developed for using discrete fractional derivative at time t_n in which some new coefficients $(k+\frac{1}{2})^{2-\alpha}-(k-\frac{1}{2})^{2-\alpha}$ instead of $(k+1)^{2-\alpha}-k^{2-\alpha}$ are derived. Stability and convergence of the method are rigourously established. We prove that the novel discretization is unconditionally stable, and the optimal convergence orders $O(\tau^{3-\alpha}+h^2)$ both in L_2 and L_∞ are derived, where τ is the time step and h is space mesh size. Moreover, the applicability and accuracy of the scheme are demonstrated by numerical experiments to support our theoretical analysis.

Keywords:
Fractional derivative, Diffusion-wave equation, Novel finite difference, Numerical experiments, Estimates
2010 MSC: 65M06, 65M12, 65M15, 26A33

1. Introductions

In recent years, many problems in physical science, electromagnetism, electrochemistry, diffusion and general transport theory can be solved by the fractional calculus approach, which gives attractive applications as a new modeling tool in a variety of scientific and engineering fields. Roughly speaking, the fractional models can be classified into two principal kinds: space-fractional differential equation and time-fractional one. Numerical methods and theory of solutions of the problems for fractional differential equations have been studied extensively by many researchers which mainly cover finite element methods [1–4], mixed finite element methods [5–8], finite difference methods [9–13], finite volume (element) methods [14, 15], (local) discontinuous Galerkin (L)DG methods[16], spectral methods [17–21] and so on.

Roughly speaking, a fractional diffusion-wave equation is a linear integro-partial differential equation obtained from the classical diffusion or wave equation by replacing the first- or second-order time derivative by a fractional derivative of order $\alpha > 0$ [22]. For $1 < \alpha < 2$, the fractional equation with initial and boundary value is expected to interpolate the diffusion equation and the wave equation, thus it is referred to as the time fractional diffusion-wave equation. In recent years, many published papers are devoted to numerical methods for solving the time fractional diffusion-wave equation. Huang et al. [12] constructed two one order finite difference schemes to solve a class of initial-boundary value time fractional diffusion-wave equations based on its equivalent partial integro-differential equations. Sun and Wu [22] developed a fully discrete difference scheme for a diffusion-wave system by introducing two new variables to transform the original equation into a low order systems equations, and gave the error analysis. This scheme was cited in many other papers [23–29] in the following several years. In this article, we give a novel fully discrete difference scheme for the fractional diffusion wave equation which is different from the Sun's scheme in [22] and prove that the difference scheme is uniquely

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