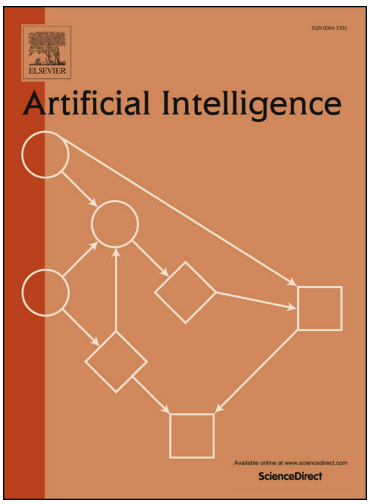
Accepted Manuscript



Approximating Optimal Social Choice under Metric Preferences

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PII: S0004-3702(18)30456-9
DOI: https://doi.org/10.1016/j.artint.2018.07.006
Reference: ARTINT 3090

To appear in: *Artificial Intelligence*

Received date: 24 March 2017
Revised date: 22 July 2018
Accepted date: 24 July 2018

Please cite this article in press as: E. Anshelevich et al., Approximating Optimal Social Choice under Metric Preferences, *Artif. Intell.* (2018), https://doi.org/10.1016/j.artint.2018.07.006

Approximating Optimal Social Choice under Metric Preferences*

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August 16, 2018

Abstract

We consider voting under metric preferences: both voters and alternatives are associated with points in a metric space, and each voter prefers alternatives that are closer to her to ones that are further away. In this setting, it is often desirable to select an alternative that minimizes the sum of distances to the voters, i.e., the utilitarian social cost, or other similar measures of social cost. However, common voting rules operate on voters' preference rankings and therefore may be unable to identify an optimal alternative. A relevant measure of the quality of a voting rule is then its *distortion*, defined as the worst-case ratio between the performance of an alternative selected by the rule and that of an optimal alternative. Thus, distortion measures how good a voting rule is at approximating an alternative with minimum social cost, while using only ordinal preference information. The underlying costs can be arbitrary, implicit, and unknown; our only assumption is that they form a metric space. The goal of our paper is to quantify the distortion of well-known voting rules. We first establish a lower bound on the distortion of any deterministic voting rule. We then show that the distortion of positional scoring rules cannot be bounded by a constant, and for several popular rules in this family distortion is linear in the number of alternatives. On the other hand, for Copeland and similar rules the distortion is bounded by a factor of 5. These results hold both for the sum of voters' cost and the median voter cost. For Single Transferable Vote (STV), we obtain an upper bound of $O(\ln m)$ with respect to the sum of voters' costs, where m is the number of alternatives, as well as a lower bound of $\Omega(\sqrt{\ln m})$; thus, STV is a reasonable, though not a perfect rule from this perspective. Our results for median voter cost extend to more general objective functions.

1 Introduction

Voting rules aggregate preferences of multiple voters (also referred to as agents) over a set of available candidates (also referred to as alternatives), enabling the voters to choose an option that reflects their collective opinion. Often, voters' preferences are determined by the candidates' positions on several issues, such as the level of taxation or military spending. In this case, each voter and each candidate can be identified with a point in the *issue space*, and voters tend to prefer candidates who are close to them to the ones that are further away. This setting can be formally modeled by embedding the input election into a *metric space*, i.e., a set of points S endowed with a distance measure d: voters' preferences are consistent with this embedding if voter i prefers candidate X to candidate Y whenever $d(i, X) < d(i, Y)$. The spatial model of preferences has received a considerable amount of attention in the social

*This paper combines and extends results presented at the 29th Conference on Artificial Intelligence, AAAI-2015 (paper titled "Approximating Optimal Social Choice under Metric Preferences") and at the 31st Conference on Artificial Intelligence, AAAI-2017 (paper titled "Social Choice Under Metric Preferences: Scoring Rules and STV").