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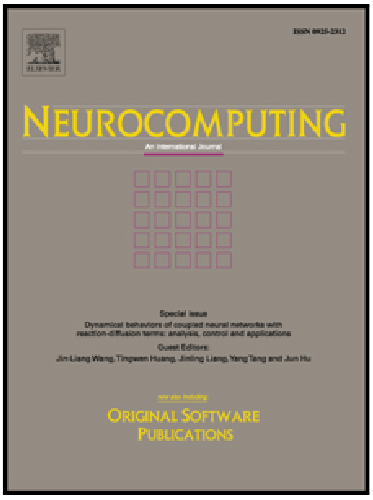

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An overview of recent developments in Lyapunov-Krasovskii functionals and stability criteria for recurrent neural networks with time-varying delays ☆

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Abstract

Global asymptotic stability is an important issue for wide applications of recurrent neural networks with time-varying delays. The Lyapunov-Krasovskii functional method is a powerful tool to check the global asymptotic stability of a delayed recurrent neural network. When the Lyapunov-Krasovskii functional method is employed, three steps are necessary in order to derive a global asymptotic stability criterion: i) constructing a Lyapunov-Krasovskii functional, ii) estimating the derivative of the Lyapunov-Krasovskii functional, and iii) formulating a global asymptotic stability criterion. This paper provides an overview of recent developments in each step with insightful understanding. In the first step, some existing Lyapunov-Krasovskii functionals for stability of delayed recurrent neural networks are anatomized. In the second step, a free-weighting matrix approach, an integral inequality approach and its recent developments, reciprocally convex inequalities and S-procedure are analyzed in detail. In the third step, linear convex and quadratic convex approaches, together with the refinement of allowable delay sets are reviewed. Finally, some challenging issues are presented to guide the future research.

Keywords: Recurrent neural networks; global asymptotic stability; time-varying delays; Lyapunov-Krasovskii functional; integral inequality.

1. Introduction

Recurrent neural networks (RNNs) were developed in the 1980s. An RNN is a network of neuron-like nodes (or called neurons), in which each neuron is connected directionally to every other neuron [1]. Neurons in an RNN can be divided into input neurons (receiving data from outside the network), output neurons (yielding results) and hidden neurons (modifying data en route from input to output). Each neuron has a time-varying real-valued activation and each directed edge (synapse) has a modifiable real-valued weight. The famous Hopfield neural network invented in 1982 [2] is a special RNN where all connections between neurons are symmetric. A defining feature of an RNN is that it can use its internal memory to process arbitrary sequences of inputs. This feature makes an RNN applicable to several tasks including unsegmented, speech recognition and solving optimization problems [3–6].

Global asymptotic stability (GAS) is an important issue for RNNs since most applications of them are heavily dependent on their stability behaviours [7–11]. For instance, when applied to signal processing, parallel computation and some optimization problems, the RNN is needed to have a unique equilibrium point that is GAS [6]. However, an RNN may have a number of equilibrium points in nature, which possibly lead to spurious suboptimal responses [12]. In this sense, the study on GAS of a unique equilibrium point for the RNN concerned is of great importance from both theoretical and applicable perspectives [6]. On the other hand, as explicitly pointed out in [13], in real RNNs and their hardware implementations, it is impossible for neurons to respond and communicate simultaneously since time-delays always exist. Notice that time-delays are usually a key factor to render an RNN unstable even though they are very small. Therefore, investigating GAS of RNNs with time-delays is important before applied. During the last decade, a great deal of results on GAS for RNNs with time-delays have been reported, see, e.g. [6, 14–20] and the references therein.

The early research on GAS of RNNs is based on some algebraic analysis methods, such as M-matrix [21], algebraic inequality [22], matrix norm [23], matrix measure [24] and spectral radius [25]. Usually, GAS criteria derived from these methods are *delay-independent* since no information on time-delays is included. These criteria can build some certain relationship between system parameters, thereby they are meaningful both in practice and in theory [6]. When information on time-delays is unknown, delay-independent stability criteria are useful to check the GAS of RNNs with time-delays. However, when information on delay bounds and delay-derivative is available, delay-independent stability criteria are conservative [27–31], especially for RNNs with *small* time-delays [32–34]. In this situation, the study of delay-independent stability should give way to the study of delay-dependent stability.

The delay-dependent GAS analysis of delayed RNNs aims at deriving an admissible delay upper bound (ADUB) such that

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