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Equilibrated warping: Finite element image registration with finite strain equilibrium gap regularization[☆]

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ABSTRACT

In this paper, we propose a novel continuum finite strain formulation of the equilibrium gap regularization for image registration. The equilibrium gap regularization essentially penalizes any deviation from the solution of a hyperelastic body in equilibrium with arbitrary loads prescribed at the boundary. It thus represents a regularization with strong mechanical basis, especially suited for cardiac image analysis. We describe the consistent linearization and discretization of the regularized image registration problem, in the framework of the finite elements method. The method is implemented using FEniCS & VTK, and distributed as a freely available python library. We show that the equilibrated warping method is effective and robust: regularization strength and image noise have minimal impact on motion tracking, especially when compared to strain-based regularization methods such as hyperelastic warping. We also show that equilibrated warping is able to extract main deformation features on both tagged and untagged cardiac magnetic resonance images.

1. Introduction

Image processing, in particular image registration for motion tracking, is playing an important role in biomedical imaging (Tobon-Gomez et al., 2013; Sotiras et al., 2013) and in other domains such as materials and mechanical engineering (Sutton and Hild, 2015). However, despite important progress made in the past decades, robustness, efficiency and precision of the existing methods must still be improved to translate them into medical and engineering applications. In this paper we propose a novel regularization approach that has a strong mechanical basis, and apply it to finite element-based image registration problems. We illustrate our approach on cardiac motion tracking from magnetic resonance (MR) images.

MR imaging (MRI) is a powerful tool that can be used to quantify the motion of the beating heart *in vivo* and non-invasively, with wide ranging clinical applications. They include the diagnosis of coronary artery diseases, myocardial ischemia and infarction, non-ischemic cardiomyopathies, ventricular dyssynchrony, etc. (Shehata et al., 2009; Ibrahim, 2011). Besides diagnosing heart diseases, cardiac motion tracking is also used as a component of other MRI techniques, for instance in *in vivo* diffusion tensor imaging (Stoeck et al., 2015; von Deuster et al., 2015), which has high clinical relevance. In the field of personalized computational modeling (Krishnamurthy et al., 2013; Lee et al., 2014), cardiac motion tracking is used to estimate model parameters that could serve as biomarkers for cardiovascular diseases (Sermesant et al., 2006; Imperiale et al., 2011).

Since regular anatomical cine MR images have little contrast within the myocardial wall, tagged MRI was designed to track material points through the generation and imaging of a magnetization grid (SPAtial Modulation of Magnetization, SPAMM) (Zerhouni et al., 1988; Axel and Dougherty, 1989). It was later improved with the Complementary SPAMM (CSPAMM) method, which prevents tag fading (Fischer et al., 1993). Accelerated whole-heart 3D sequences (3D CSPAMM) have since been proposed (Ryf et al., 2002), which mitigate misregistration issues common with multi-slice acquisitions and allow for fast and reliable tracking of the entire left ventricle throughout the cardiac cycle (except for end-diastole) in

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only three (Rutz et al., 2008), or even a single (Stoeck et al., 2012) breath hold.

Multiple approaches have been proposed to post-process cardiac magnetic resonance images and extract motion and deformation fields (Wang and Amini, 2012; Tobon-Gomez et al., 2013). They vary in the nature of the *a priori* knowledge that is used to better distinguish signal from noise, and how it is incorporated. A first distinction exists between data assimilation and image registration techniques. In data assimilation, a realistic physical model is used and its parameters are estimated to best match the acquired images (Sermesant et al., 2006; Sainte-Marie et al., 2006). This estimation is either variational (Delingette et al., 2012) or sequential (Moireau et al., 2008; Chapelle et al., 2013). In other communities, this is called integrated image correlation (Hild and Roux, 2006; Hild et al., 2016). Conversely, in image registration techniques, which are the focus of this paper, only limited *a priori* knowledge is required, solely to regularize the registration problem. The strengths and weaknesses of both approaches are opposite: using a realistic model as regularizer allows to process low resolution and/or noisy data while extracting meaningful physical parameters; on the other hand the processing is highly dependent on the validity of the model.

Within image registration methods, another distinction exists, between Fourier-based and tracking-based methods (Tobon-Gomez et al., 2013). Among Fourier-based methods, HARmonic Phases (HARP)-based techniques (Osman et al., 1999; Garot et al., 2000) are the most commonly used methods, which have enabled fully automatic post-processing of tagged images. Based on similar concepts, the SinMod method (Arts et al., 2010; Wang et al., 2013) offers improved motion tracking performance. However, these methods are limited to tagged images, as HARP requires phase data, and SinMod makes use of the sine modulation of magnitude data. In contrast, tracking-based methods, which are the focus of this paper, can be applied to any type of images.

A final notable distinction exists, within tracking-based image registration techniques, between local and global approaches (Hild and Roux, 2012). In local approaches, images are correlated region by region (Lenoir et al., 2007), while in global approaches they are correlated at once (Veress et al., 2005; Phatak et al., 2009). Note that hybrid methods have been proposed, which efficiently alternate between local and global steps (Thirion, 1998; Vercauteren et al., 2008).

The present paper focuses on global tracking-based image registration, specifically on the regularization of the registration problem. Basic mathematical regularization such as Laplacian smoothing (Passieux and Périé, 2012) penalizes rigid body rotations and are thus limited to very small deformations. Fluid-like mechanical regularization has been proposed in Christensen et al. (1996), which is not suitable for the (solid) myocardium as it does not enforce the continuity of the displacement field. Incompressibility has also been used as a regularizer (Mansi et al., 2011; McLeod et al., 2012), but it represents a strong kinematic constraint that can potentially interfere with the estimation of the actual kinematics described in the images. Hyperelastic regularization (Veress et al., 2005; Phatak et al., 2009; Burger et al., 2013) uses a proper strain measure valid for arbitrary large rotations and deformations, but still penalizes strain itself, hence might be considered too strong of a regularization as well.

In this paper, we propose a novel regularizer for finite element-based image registration problems based on the continuum finite strain formulation of the equilibrium gap principle. This regularizer has strong physical basis as it penalizes any deviation from the mechanical equilibrium (which is a fundamental principle) instead of penalizing the kinematics itself (which can be arbitrary). It also benefits from the finite elasticity framework, and does not penalize rigid body displacement or rotation. This work inherits ideas from (Claire et al., 2004; Leclerc et al., 2010), where a similar regularizer has been formulated at the discrete level, and within the linearized elasticity framework. The formulation is presented in details in Section 2, validation is provided in Section 3.1, and an illustration on *in vivo* cardiac MR images is given in Section 3.2.

2. Methods

2.1. Finite element-based image registration

2.1.1. Problem

Formulation. Let us denote I_0 & I_t as the scalar intensity fields of two images representing an object occupying the domains Ω_0 & Ω_t in the reference and deformed states, respectively. The problem is to find the smooth mapping $\underline{\varphi}$, or equivalently the smooth displacement field $\underline{U}$ ($\underline{\varphi}(\underline{X}) = \underline{X} + \underline{U}(\underline{X})$), between Ω_0 & Ω_t. This problem is ill-posed, notably because of image noise, and must thus be formulated as a regularized minimization problem:

$$\text{find } \underline{U} = \text{argmin}_{\{\underline{U}^*\}}\left\{J(\underline{U}^*) = (1-\beta)\Psi^{\text{im}}(\underline{U}^*) + \beta\Psi^{\text{reg}}(\underline{U}^*)\right\}, \tag{1}$$

where J is the functional to minimize, Ψ^{im} the image similarity metric or "energy", Ψ^{reg} the regularization "energy", and β is the regularization strength.

Similarity metric. We use a simple sum of squares between image intensities, written here in the reference configuration, as similarity metric:

$$\begin{aligned}\Psi^{\text{im}}(\underline{U}^*) &= \frac{1}{2}\int_{\Omega_0}\left(I_t(\underline{X}+\underline{U}^*(\underline{X})) - I_0(\underline{X})\right)^2 d\Omega_0 \\ &= \frac{1}{2}\int_{\Omega_0}\left(I_t\circ\underline{\varphi}^* - I_0\right)^2 d\Omega_0.\end{aligned} \tag{2}$$

Regularization. Many regularizers have been proposed for image registration problems, including fluid (Christensen et al., 1996) and hyperelastic (Veress et al., 2005; Phatak et al., 2009; Genet et al., 2016) constraints.

In hyperelastic warping (Veress et al., 2005; Phatak et al., 2009), the regularization energy is directly the strain energy of the body:

$$\Psi^{\text{reg,hyper}} = \int_{\Omega_0}\rho_0\psi\, d\Omega_0, \tag{3}$$

where ρ_0 is the mass density, and ψ the specific strain energy potential. Thus, strain, a quantity that we seek to extract from the images and that can be in principle arbitrary, is directly penalized by the regularization in hyperelastic warping. Moreover, the minimizer of Problem (1–3), being the minimum of a "potential" energy, is the solution of a mechanical problem of a body in equilibrium under an unphysical body force that corresponds to the mismatch between the image intensity fields. The obtained deformation therefore has weak mechanical basis.

Motivated by this inconsistency, we propose an alternate regularizer, which essentially penalizes any deviation from the solution of a hyperelastic body in equilibrium with arbitrary boundary loads (but no body load). Let us first recall that mechanical equilibrium, i.e., conservation of momentum, in absence of body load and inertia, can be expressed as:

$$\begin{cases}\underline{\text{Div}}\left(\underline{\underline{F}}\cdot\underline{\underline{S}}\right) = 0 \\ {}^t\underline{\underline{S}} = \underline{\underline{S}}\end{cases} \quad \forall \underline{X}\in\Omega_0, \tag{4}$$

where $\underline{\underline{S}}$ is the second Piola–Kirchhoff stress tensor, and $\underline{\underline{F}} = \frac{\partial\underline{\varphi}}{\partial\underline{X}}$ the transformation gradient (Holzapfel, 2000). These relations correspond to conservation of linear and angular momentum, respectively. We also recall that the second principle of thermodynamics