Swarm and Evolutionary Computation xxx (2018) 1–19

Contents lists available at ScienceDirect

Swarm and Evolutionary Computation

journal homepage: www.elsevier.com/locate/swevo

# Multiobjective in-core nuclear fuel management optimisation by means of a hyperheuristic

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ARTICLE INFO

*Keywords:*
Hyperheuristic
Multiobjective optimisation
In-core fuel management optimisation
Nuclear reactor

ABSTRACT

This paper is concerned with the problem of constrained *multiobjective in-core fuel management optimisation* (MICFMO) using, for the first time, a *hyperheuristic* technique as solution approach. A multiobjective hyperheuristic called the AMALGAM method (an evolutionary-based technique incorporating multiple sub-algorithms simultaneously) is compared to three previously-studied metaheuristics, namely the *nondominated sorting genetic algorithm II*, the *Pareto ant colony optimisation* algorithm and the *multiobjective optimisation using cross-entropy method*, in an attempt to improve upon the level of generality at which MICFMO may be conducted. This solution approach was motivated by a lack of consistent performance by the aforementioned metaheuristics when applied in isolation. Comparisons are conducted in the context of a test suite of several problem instances based on the SAFARI-1 nuclear research reactor. Nonparametric statistical analyses in respect of the optimisation results reveal that the AMALGAM method significantly outperforms the three metaheuristics in the majority of problem instances within the test suite. Additional comparisons are also performed between the proposed AMALGAM method and a randomised (or no-learning) version thereof, as well as a selection choice function-based multiobjective hyperheuristic available in the literature. It is found that the proposed method is superior to the choice function-based algorithm within the context of the MICFMO test suite, and yields results of similar quality when compared to its randomised version. The practical relevance of the hyperheuristic results is further demonstrated by comparing the solutions thus obtained to a reload configuration designed according to the current fuel assembly reload design approach followed at the SAFARI-1 reactor.

## 1. Introduction

The *in-core fuel management optimisation* (ICFMO) problem refers to the task of finding an optimal assignment of fuel assemblies to loading positions in a nuclear reactor core, subject to certain constraints. Such an assignment of assemblies is referred to as a *fuel reload configuration* (or a *loading pattern*). The ICFMO problem is well known in the field of nuclear engineering and has been a subject of research for many years [1–4]. Characteristics associated with this problem include its large combinatorial decision space, multiple conflicting, nonlinear objective functions and constraints that generally cannot be expressed in closed form, and computationally expensive function evaluations using a reactor core simulator [5,6].

A number of solution techniques within the realms of mathematical programming, expert- or knowledge-based systems and metaheuristics, have been proposed for solving the ICFMO problem [2,5]. Metaheuristics have, in particular, emerged as the most prominent solution techniques applicable to the problem. Examples thereof include simulated annealing [4], genetic algorithms [7], particle swarm optimisation [8], ant colony optimisation [1] and tabu search [9]. Apart from solution techniques, research efforts have also been aimed toward reducing the computational cost associated with function evaluations in the ICFMO problem. In this regard, *artificial neural network* (ANN) surrogate models have been applied with good effect for predicting objective function and constraint function values, as opposed to calculating them explicitly using reactor core simulators [10,11]. Incorporating such surrogate models within computationally expensive optimisation procedures is a

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https://doi.org/10.1016/j.swevo.2018.02.019
Received 7 December 2016; Received in revised form 7 November 2017; Accepted 24 February 2018
Available online XXX

relatively active field of research and the reader is referred to [12] for further information on this topic.

It has been pointed out that, very often, multiple objective functions have to be optimised simultaneously in instances of the ICFMO problem [5,6]. The overwhelming majority of research concerning ICFMO has, however, been performed in the context of single-objective optimisation. In multiobjective optimisation, the aim is to identify a set of trade-off solutions to a problem instance, based on the fundamental notion of Pareto optimality. Accordingly, the *multiobjective in-core fuel management optimisation* (MICFMO) problem is the problem of finding a so-called Pareto optimal set of fuel reload configurations for a nuclear reactor core.

Until recently, only a few *multiobjective optimisation algorithms* (MOAs) had been applied to the MICFMO problem in the literature. Multiobjective genetic algorithm approaches were considered by Parks [3], Do and Nguyen [13], and Hedayat *et al.* [14], whereas multiobjective simulated annealing approaches were considered by Park *et al.* [15] and Engrand [16]. Improvements on Engrand's simulated annealing algorithm were also proposed by Parks and Suppapitnarm [17] as well as by Kellar [18]. Finally, a multiobjective particle swarm optimisation algorithm was applied to the MICFMO problem by Babazadeh *et al.* [19], while Schlünz *et al.* [20] considered a multiobjective cross-entropy method. In each case, however, the aforementioned MOAs were employed in isolation — their efficacy and comparative performance in respect of solving the MICFMO problem therefore remained unknown. Then, in 2016, Schlünz *et al.* [21] conducted an investigation in which eight modern state-of-the-art MOAs were compared for solving several constrained MICFMO problem instances in a test suite based on the SAFARI-1 research reactor. In that study, it was found that the *nondominated sorting genetic algorithm II* (NSGA-II) [22], the *Pareto ant colony optimisation* (P-ACO) algorithm [23] and the *multiobjective optimisation using the cross-entropy method* (MOOCEM) [24] were generally the best-performing metaheuristics for solving constrained instances of the MICFMO problem. No single MOA was, however, able to consistently outperform the other metaheuristics with respect to all, or most of, the MICFMO problem instances in the test suite.

The aforementioned lack of consistent MOA performance in the context of MICFMO therefore motivates further studies into the suitability of different solution techniques for the problem. Accordingly, this paper builds upon the work of Schlünz *et al.* [21] by attempting to improve the level of generality at which constrained MICFMO may be performed. This is achieved by investigating, for the first time, the efficacy of a multiobjective *hyperheuristic* solution technique known as the AMALGAM method (which is an acronym for *a multi-algorithm, genetically adaptive multiobjective*) [25]. Hyperheuristics have been shown not only to raise the level of general applicability, but also to achieve improved quality and/or efficiency in optimisation results [26]. In the case of the AMALGAM method, which incorporates multiple metaheuristics simultaneously in an adaptive manner, testing of the method by Vrugt and Robinson [25] in respect of benchmark problem instances in the continuous domain revealed that improvements in computational efficiency of up to a factor of ten may be achieved over the individual sub-algorithm metaheuristics.

The same test suite of constrained MICFMO problem instances considered in Ref. [21] is employed in this paper, along with corresponding ANN surrogate models for faster function evaluations [27]. Using this test suite, the AMALGAM method is compared to the NSGA-II, the P-ACO algorithm and the MOOCEM in a structured and statistically sound manner, as advocated in the literature [28,29]. These three metaheuristics are also employed as sub-algorithms in the AMALGAM method. Furthermore, in order to test the efficacy of the learning mechanism within the proposed approach, the AMALGAM method is compared to a randomised (or no-learning) version thereof. Similarly, the method is also compared to another multiobjective hyperheuristic recently proposed in the literature. Both these comparisons are performed within the context of the aforementioned MICFMO test suite. Finally, the non-parametric Friedman test, the Nemenyi, Wilcoxon-Wilcox, Miller *post hoc* procedure, and the Wilcoxon signed rank test, in particular, are employed within the statistical comparative analysis [30].

The paper is organised as follows. The constrained MICFMO problem, the test suite of problem instances and the corresponding ANN surrogate models are described in §2. A brief introduction to hyperheuristics is then presented in §3, along with a description of the AMALGAM method and its implementation for constrained MICFMO. Thereafter, the performance assessment and statistical testing procedures adopted in this study are discussed in §4. The various comparative results obtained in respect of the test suite are then presented and analysed in §5. The practical relevance of the AMALGAM method is also demonstrated in §6 by comparing a subset of the results obtained with that rendered by the current SAFARI-1 reload configuration design approach. The paper finally closes in §7 with a brief set of conclusions.

## 2. The constrained MICFMO problem

The MICFMO problem may generally be regarded as a *nonlinear assignment problem* in which available fuel assemblies are to be assigned in an optimal manner to loading positions in a nuclear reactor core for a particular operational cycle of the reactor. Although the problem may be extended so as to include multiple operational cycles, only the *single-cycle* version thereof is considered in this paper. The optimisation model for the constrained MICFMO problem proposed in Ref. [21] is also adopted in this work. Accordingly, it is assumed that the number, $n$, of fuel assemblies (labelled $1, \ldots, n$) is equal to the number of loading positions in the core (also labelled $1, \ldots, n$). In the model, a reload configuration is represented by a permutation vector $\boldsymbol{x} = [x_1, \ldots, x_n]$ where the decision variable $x_i = j$ denotes that fuel assembly $j \in \{1, \ldots, n\}$ is assigned to loading position $i \in \{1, \ldots, n\}$. Furthermore, $\mathcal{X}$ is the set of all possible reload configurations (*i.e.* permutation decision vectors). All objective functions are also assumed, without loss of generality, to require maximisation. The general form of the constrained MICFMO problem with $q$ objective functions $f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_q(\boldsymbol{x})$ is therefore formulated in Ref. [21] as

$$\left.\begin{array}{lll} \text{maximise} & \boldsymbol{f}(\boldsymbol{x}) = [f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_q(\boldsymbol{x})], & \\ \text{subject to} & g_i(\boldsymbol{x}) \leq g_i^{\lim}, & i = 1, \ldots, r, \\ & h_j(\boldsymbol{x}) = h_j^{\lim}, & j = 1, \ldots, s, \\ & \boldsymbol{x} \in \mathcal{X}, & \end{array}\right\} \quad (1)$$

where $g_i(\boldsymbol{x})$ and $g_i^{\lim}$ are the inequality constraint functions and their corresponding (non-zero) limiting values, respectively, for $i = 1, \ldots, r$. Similarly, $h_j(\boldsymbol{x})$ and $h_j^{\lim}$ are the equality constraint functions and their corresponding (non-zero) values, respectively, for $j = 1, \ldots, s$.

### 2.1. The test suite for constrained MICFMO

The test suite of constrained MICFMO problem instances proposed by Schlünz *et al.* [21] is employed in this paper. These instances are based on the SAFARI-1 nuclear research reactor in South Africa, which is primarily utilised for commercial irradiation services, along with nuclear and materials research. The commercial services rendered by the reactor revolve around the production of radioisotopes (primarily molybdenum-99, which is used for medical diagnostic purposes) and the neutron transmutation doping of silicon (to produce silicon semiconductors used in electronic equipment). Several neutron beam tubes surrounding the reactor core, are utilised for neutron scattering, radiography and diffraction experiments in respect of the nuclear and materials research.

The core layout of the SAFARI-1 reactor is presented in Fig. 1. Currently, the reactor core consists of a $9 \times 8$ lattice which houses twenty-six fuel assemblies, six control rods, seven dedicated molybdenum pro-