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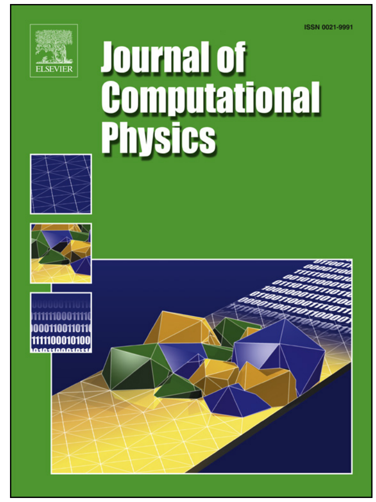

The finite difference/finite volume method for solving the fractional diffusion equation

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Highlights

- In this paper, we consider the initial-boundary problem of the time-fractional diffusion equation:

$$\begin{cases} \partial_t^\alpha u - \text{div}(A\nabla u) = f(t,x),\ in\ \Omega,\ 0 < t \le T, \\ u = 0,\ \ on\ \partial\Omega,\ 0 < t \le T, \\ u(0,x) = u_0(x),\ x \in \Omega, \end{cases} \tag{0.1}$$

where ∂_t^α denotes the time-fractional differential operator of Caputo type with order $0 < \alpha < 1$, $\Omega \subset R^2$ is a convex polygonal domain with boundary $\partial\Omega$, $A = (a_{ij}(x))_{2\times 2}$ is the coefficient matrix and $T > 0$ is a fixed value.

We present and analyze a fully discrete numerical scheme which is based on the linear finite volume method for the spatial discretization and the $L1$ difference approximation to $\partial_t^\alpha u$. The main contribution of this paper is as follows.

1 We establish a new error bound of $O(\Delta t^{1+\delta-\alpha})$-order for the $L1$ formula under the condition of $u(t) \in C^{1,\delta}[0,T]$ where $0 \le \delta \le 1$ is the Hölder continuity index.

2 We prove that this fully discrete finite volume scheme is unconditionally stable and the discrete solution admits the optimal error estimate of $O(\Delta t^{1+\delta-\alpha} + h^2)$-order in the $L2$ norm.

We emphasize that few finite volume research works can be found in existing literatures for solving the time-fractional diffusion equations.