Journal of Computational Physics 375 (2018) 337–351

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

An approximate Jacobian nonlinear solver for multiphase flow and transport

Gurpreet Singh *, Gergina Pencheva, Mary F. Wheeler

Institute for Computational Engineering and Sciences, The University of Texas at Austin, United States of America

ARTICLE INFO

Article history:
Received 28 November 2017
Received in revised form 22 August 2018
Accepted 24 August 2018
Available online 29 August 2018

Keywords:
Nonlinear solver
Jacobian approximation
Mixed finite elements
Cell-centered finite difference
Fully-implicit

ABSTRACT

We present an approximate Jacobian approach for solving nonlinear, multiphase flow and transport problems in porous media. A backward Euler time discretization scheme is used prior to spatial discretization with a lowest order mixed finite element method (MFEM). This results in a fully implicit nonlinear algebraic system of equations. Conventionally, an exact Jacobian construction is employed during the Newton linearization to obtain a linear system of equations after spatial and temporal discretization. This fully coupled, monolithic linear system, usually in pressure and saturation (or concentration) unknowns, requires specialized preconditioners such as constrained pressure residual (CPR) or two stage preconditioner. These preconditioners operate on the linear system to decouple pressure and saturation (or concentration) degrees of freedom (DOF) in order to use existing linear solvers for positive definite (PD) matrices such as GMRES or AMG, to name a few. In this work, we present an alternative to two-stage preconditioning (or CPR) for solving the aforementioned monolithic system after Newton linearization. This approach relies upon a decoupling approximation for the pressure-saturation (or concentration) block sub-matrices, during Newton linearization, to obtain block diagonal sub-matrices. The resulting linear system is easily reduced, trivially eliminating these diagonal sub-matrices, to obtain a system in pressure DOF circumventing the need for specialized preconditioners. Further, the linear system has lesser DOF owing to the elimination of saturation (or concentration) unknowns. This nonlinear solver is demonstrated to be as accurate as the exact Jacobian approach, measured in terms of convergence of nonlinear residual to a desired tolerance for both methods. Our numerical results indicate a consistent computational speedup by a factor of approximately 1.32 to 4.0 for the two-phase flow model formulation under consideration. This is related to the DOF of the linear systems for the approximate and exact Jacobian approaches. For multicomponent flow and transport, this speedup is expected to be directly proportional to the number of concentration degrees of freedom. A number of field scale numerical simulations are also presented to demonstrate the efficacy of this approach for realistic problems.

Published by Elsevier Inc.

* Corresponding author.
E-mail address: gurpreet@utexas.edu (G. Singh).

https://doi.org/10.1016/j.jcp.2018.08.043
0021-9991/Published by Elsevier Inc.

1. Introduction

Nonlinear processes are a common occurrence for multiphase, flow and transport problems in sub-surface porous media. These nonlinearities originate from empirical, phenomenological and often mechanistic considerations during the modeling of the physical processes. Conventionally, after spatial and temporal discretization of the partial differential equations of the associated model formulation, linearization is necessary to solve the resulting nonlinear system of algebraic equations. The Newton's method is a well-known nonlinear solver and has proven quadratic rate of convergence in the vicinity of the solution assuming a Lipschitz continuity argument. There is an extensive literature dedicated to different modifications of Newton's method that address a wide array of issues in solving nonlinear system of equations (inexact, chord, quasi-Newton, semismooth, etc.), see e.g. [1–6] and references therein. The semi-smooth Newton methods, for example, are used for treating system of non-smooth equations with lower regularity arising from model inequality constraints, inadvertent roughness due to empirical considerations, or other modeling inconsistencies. This latter inconsistency is often seen in realistic subsurface reservoirs with heterogeneous rock properties, where an empirical description of capillary pressure and relative permeability introduces discontinuities at the interface between different rock types.

For flow and transport problems in porous media, a backward Euler discretization in time with an appropriate spatial discretization gives rise to a fully-implicit nonlinear system of algebraic equations. A nonlinear solver, as described before, is then used to obtain spatial distributions of desired unknowns (pressure, saturation, concentration etc.) at a given time. In doing so, the solution available at the previous time, the time-step size, and the nonlinear solver convergence rates are inherently tied. The primary focus in the development of computationally efficient, nonlinear solution strategies is to either increase the convergence rates (optimally quadratic) or increase time-step sizes although at the cost of additional numerical diffusion. However, these two desirable properties are closely related due to the linearization assumption inherent to the nonlinear solver. A large time-step size requirement strains this linearization assumption, since an initial estimate from previous time solutions is not sufficiently close to the final solution, resulting in reduced convergence rates. Similarly, a requirement on the convergence rate constraints the time-step size. Since a rigorous derivation relating time-step size and nonlinear convergence rate is not often achievable in the light of the wide ranging model nonlinearities, we draw our conclusions based upon observed numerics.

Several approaches have been proposed that aim to alleviate or circumvent some of these aforementioned issues in favor of overall computational efficiency. A number of these approaches [7–10] improve nonlinear convergence rates, for a given time-step size, that rely upon modifying the Newton step-size or descent direction or both. These modifications are either based upon determining an optimal nonlinear step size using line-search algorithms or altering descent direction (Jacobian) using prior knowledge of the regularity of functions contributing to the Jacobian. The simplest example of modifying the descent direction occurs in the case of slightly compressible fluid description where the contribution of the density derivative with respect to pressure to the Jacobian is often considered negligible. This latter modification improves the overall efficiency by neglecting the computationally expensive evaluation of the density derivative. Another such approach, is the reduced Newton algorithm proposed by [9,10] that relies upon saturation or concentration updates in an order determined by pressure potential. The authors report an overall reduction in computational cost due to larger time-step sizes for which the nonlinear iterations converge. However, the sequential nature of potential reordering and consequent saturation/concentration updates in this proposed nonlinear solution algorithm might pose parallel scalability issues.

In this work, we present a nonlinear solver based upon modifying the Newton descent direction to improve the overall computational efficiency for numerical reservoir simulations. Although, this work has been extended for fully implicit, multiphase, compositional flow [11], the model complexity precludes a fair comparison between the proposed and conventional approaches. We therefore restrict ourselves to a slightly compressible, two phase flow model for the sake of simplicity. This allows us to compare the differences between the conventional Newton method (exact Jacobian) and our approach (approximate Jacobian) both in terms of nonlinear system formulation and consequent numerical benchmarking. We begin with a description of a two-phase flow model formulation followed by a brief discussion of a mixed finite element spatial discretization and its relation to the well known cell-centered finite difference. We then present a δ notation relying upon Gâteaux derivatives to linearize the semi-discrete nonlinear partial differential equations obtained after temporal discretization prior to spatial discretization. This is followed by a detailed discussion on the conventional and proposed approaches. We use this aforementioned δ notation to simplify the description and easily distinguish the differences in Jacobian construction for the two approaches. Next we describe the nonlinear solvers and preconditioners used in this work with a brief discussion on the accuracy of the two nonlinear solvers. Finally, we present extensive numerical experiments for benchmarking the proposed approach against the conventional method to determine the computational speedup and overall increase in efficiency.

2. Model formulation

We begin by describing the model formulation for immiscible, two-phase, slightly compressible flow in a porous medium which is widely accepted in several porous media communities such as oil and gas, ground water hydrology and environmental engineering.