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Transition to a chaotic flow in a V-shaped triangular cavity heated from below

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Natural convection in a V-shaped cavity heated from below and cooled from top is investigated owing to its extensive presence in industrial systems and in nature such as in a valley. Two dimensional numerical simulation is performed for natural convection in the cavity using a Finite Volume Method. A wide range of Rayleigh numbers of $Ra = 10^0$ to 10^8 for the aspect ratio of $A = 0.5$ and the Prandtl number of $Pr = 0.71$ is considered. A set of supercritical bifurcations in a transition to a chaotic flow are described, which include a Pitchfork bifurcation from symmetric to asymmetric state and a Hopf bifurcation from steady to unsteady state. It is found that the Pitchfork bifurcation occurs between $Ra = 7.5 \times 10^3$ and 7.6×10^3 and the Hopf bifurcation occurs between $Ra = 1.5 \times 10^7$ and 1.6×10^7. Additionally, a further bifurcation from periodic to chaotic state occurs between $Ra = 5 \times 10^7$ and 6×10^7. The power spectral density, the phase space trajectory and the largest Lyapunov exponent of unsteady flows in the transition to a chaotic state have been described. Further, heat transfer in the cavity is calculated and the corresponding dependence on the Rayleigh number is discussed and quantified.

1. Introduction

Free or natural convection is easily found in nature and industrial systems and thus is receiving continuing attention [1]. In particular, free convection in an enclosure like square or rectangular shape has been investigated by many investigators owing to the simple geometry [2,3].

Natural convection in an enclosure usually involves two scenarios for which the cavity is imposed by a vertical temperature gradient and by a horizontal temperature gradient. One of the earliest studies of natural convection in the cavity imposed by a horizontal temperature difference was reported by Batchelor [4], which demonstrated that the mode of heat transfer is primarily dominated by conduction for sufficiently small Rayleigh numbers. However, natural convection flows are dominated by convection if the Rayleigh number exceeds the critical value. Steady natural convection flows were focused in many early studies [5–7]. An interesting fact is that natural convection flows at the primary symmetric state in the cavity are induced through the viscous shear by the baroclinity, which is generated by the thermal vertical wall [8]. The thermal boundary layer adjacent to the vertical wall is steady for small Rayleigh numbers, but distinct travelling waves appear in the thermal boundary layer owing to convective instability for sufficiently large Rayleigh numbers [9]. As the Rayleigh number increases, natural convection flows can be transitional or even turbulent [10–12]. In addition, transient natural convection flows after sudden heating are also recently investigated and the corresponding dynamics is discussed [8,13] in which the feature of transient natural convection flows is characterized. Apart from natural convection flows driven by the baroclinity, natural convection flows induced by a vertical temperature difference, termed as Rayleigh-Bénard instability, are also extensively present and considerably studied [14]. Particularly, turbulent Rayleigh-Bénard convection in the cavity is recently focused by increasing investigators [15].

A square or rectangular cavity is not an adequate model for many industrial systems and geophysical situations in which the cavity geometry varies or involves one or more inclined walls. Natural convection flows on an inclined surface, termed by 'anabatic flow' or 'katabatic flow', are also common in industrial systems and in nature such as in a valley and thus have been paid considerable attention [16,17]. Naturally, natural convection flows

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Nomenclature

A	aspect ratio (H/L)
f_p	non-dimensional peak frequency
g	gravitational acceleration (m/s^2)
Gr	Grashof number, $g\beta(T_h - T_c)H^3/\nu^2$
H, L	height and half length of the cavity (m)
I	degree of symmetry
ln	non-dimensional length of the inclined wall (l/H)
n	non-dimensional coordinate normal to the inclined wall
N_{cells}	number of cells in the cavity
Nu	Nusselt number
P	pressure (N/m^2)
p	non-dimensional pressure
Pr	Prandtl number, ν/κ
Ra	Rayleigh number, $g\beta(T_h - T_c)H^3/\nu\kappa = Gr \times Pr$
Ra_I	Critical Rayleigh number for a transition to asymmetric state
s	non-dimensional coordinate along the inclined wall
T	temperature (K)
T_0	initial temperature (K)
T_c	temperature ($T_0 - \Delta T/2$) of the cold top wall (K)
T_h	temperature ($T_0 + \Delta T/2$) of the hot inclined wall (K)
t	time (s)
ΔT	temperature difference between top and inclined walls, ($T_h - T_c$) (K)
U, V	velocity components (m/s)
U_s	velocity scale along the inclined wall at a steady state
u, v	non-dimensional velocity components in x- and y-directions
u_j	x-velocity after j time steps
X, Y	horizontal and vertical coordinates
x, y	non-dimensional horizontal and vertical coordinates
Λ_k	Kolmogorov scale of dissipative eddies
λ_k	non-dimensional Kolmogorov scale
β	coefficient of thermal expansion (1/K)
ε_0	non-dimensional distance between two points at the initial time τ_0
$\varepsilon(\tau)$	non-dimensional distance between two points at time τ
$\varepsilon(u_j)$	Euclidean distance between u_j and u_{j-1}
λ_L	largest Lyapunov exponent
κ	thermal diffusivity (m^2/s)
ν	kinematic viscosity (m^2/s)
ρ	density (kg/m^3)
θ	non-dimensional temperature
τ	non-dimensional time
τ_0	initial non-dimensional time at fully developed stage
τ_n	the non-dimensional time after n time steps from τ_0
$\Delta\tau$	non-dimensional time step

in a cavity with one or more inclined walls have been increasingly studied owing to their wide presence [18]. In fact, natural convection flows in a triangular cavity are an important extension of Rayleigh-Bénard convection [19,20]. Therefore, the dynamics and heat transfer of natural convection flows in an attic-shaped cavity imposed by an inverse temperature gradient are investigated in previous studies [21–27]. The scaling relations of natural convection flows in the cavity are obtained in [21]. Additionally, the flow structure is visualized and the corresponding heat transfer is measured in [22,23]. Further, instability and bifurcations of solutions are analyzed in [24–27]. The transition from symmetric to asymmetric flow is discussed in [28] in which a Pitchfork bifurcation is characterized with the increase of the Grashof number and validated by the experimental data. Recently, transient natural convection flows in the attic-shaped cavity have also been visualized in [29]. The development of transient flows following sudden heating and cooling is classified into three distinct stages: an initial stage, a transitional stage, and a steady or quasi-steady stage. That is, the fluid inside the boundary layer moves along the inclined wall and reaches the bottom tip in the case of cooling the inclined wall and heating the bottom surface. After that it moves toward the core of the cavity by changing its direction. Usually, the thickness of the thermal boundary layer adjacent to the inclined wall is larger than the vertical distance from the midpoint of the inclined wall to the horizontal wall for small Rayleigh numbers. As the Rayleigh number increases, the thermal boundary layer becomes stronger and separates from the inclined wall. In addition, heat transfer through the attic space subject to sudden and ramp heating or cooling boundary conditions have also been investigated in [30–34].

Studies of modelling reservoir sidearm, shallow-water heat transfer or seashores with an inclined bottom wall of a wedge-shaped triangular cavity are also conducted by many authors [35–41]. It is revealed from their studies that the development of the boundary layer flows from an isothermal and motionless state passes through several distinct stages [35–37]. In the early stage, conduction dominates heat transfer; natural convection flows become dominant in the transitional stage, which explore the existence of instabilities originating from the lowest corner termed as the rising plumes from the inclined wall; in the quasi-steady stage, the flow is described by the steadily increasing temperature and by instabilities of decreasing intensity. Based on the horizontal position of the wedge-shaped cavity the dominant mode of fluid flows and heat transfer for different Rayleigh numbers have been described in [38–40]. Three distinct flow regimes are found in the shallow littoral region for different Rayleigh number. For small values Rayleigh number, the heat transfer of the entire domain is dominated by pure conduction. As the Rayleigh number increases, heat transfer is still dominated by conduction near the shallow region, but a stable convective flow becomes the dominant mode with the increase of the distance from the shore. For larger Rayleigh numbers, heat transfer is dominated by conduction in the near shore but a small part far from the near shore shows a steady convection and the rest is dominated by unsteady convective flow. Natural convection flows in a reservoir-shaped cavity are also discussed in [41]. For different Rayleigh numbers, two distinct thermal layers are formed in the quasi-steady state: the one is an inflow along the bottom and the other is an unsteady return flow immediately under the water surface. Heat transfer in the tip region is primarily by conduction with a corresponding relatively simple flow, whereas instabilities dominate natural convection flows in the deeper region if the Rayleigh number is sufficiently large.

Natural convection flows of initially stratified fluid in a V-shaped cavity are investigated by few researchers [18,42]. Their intension is aimed at understanding of the mechanism for appearance and disappearance of fog in a valley. The experiment in [18] visualizes the breakup of the stratified structure in a V-shaped water tank. Two flow configurations are described. Firstly, when the heat flux is stronger than the effect of stratification, the upslope flow can reach the ceiling of the cavity and is horizontally discharged into the tank; secondly, if the effect of stratification becomes more important than the heat flux, the flow is mainly present adjacent to the lower-half of the inclined wall. Recently,