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Non-linear ultrasonic monitoring of damage progression in disparate rocks

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ABSTRACT

This paper focusses on non-destructive characterization of stress-induced damage progression in three types of rocks by experimentally capturing the elasto-dynamic non-linear response of the rocks, with the hypothesis that non-linear approaches have increased sensitivity relative to linear ultrasonic testing approaches. For this task, a non-linear ultrasonic testing procedure known as the Scaling Subtraction Method (SSM) has been implemented on uniaxially loaded rock specimens. The SSM procedure is based on exciting the damaged medium with two acoustic signals at different amplitudes consecutively. The linearly rescaled acoustic response at low excitation is subtracted from the response at large amplitude excitation for quantification of the elastic non-linearity of the medium by a non-linear indicator θ. At first, aluminum, Lyons sandstone, granodiorite and Gosford sandstone specimens were characterized using SSM and their inherent non-linearity was quantified. After this, the rock specimens were loaded under uniaxial compressive step-loading and ultrasonic measurements were performed simultaneously at each loading step. The non-linear indicator θ was calculated as the specimens were progressively damaged, and the changes in the non-linear response were linked to the different physical processes accompanying damage progression in rocks. The study concluded that the non-linear SSM technique is capable of sensitively recording signatures of the different stages of damage evolution in rocks.

1. Introduction

Numerous critical structures, like dams, bridges, tunnels, caverns, nuclear waste repositories, have been constructed with rocks being the ultimate load bearing material. During the design life of these structures, rocks are generally subjected to anisotropic stresses (mechanical, thermal, chemical or seismic) which can lead to formation, growth and coalescence of micro-cracks eventually causing a degradation in the rock's strength.[1–5] As a result, the life span of these structures is compromised, putting the involved life and property at risk. Therefore, it is important to consider the effect of micro-damage induced strength degradation in the engineering design and put in-place an efficient monitoring plan. Consequently, it is imperative to develop techniques through which one can monitor early damage manifestations in rocks, and at the same time understand the damage processes associated with micro-structure in rocks.[6,7] Non-linear ultrasonic testing (NLUT) techniques have shown a great potential in detecting low level of damage in a wide spectrum of materials ranging from synthetic rocks (e.g. concrete and mortar) to rocks.[8] In this study a variant of NLUT technique - the Scaling Subtraction Method (SSM) - has been used to evaluate its potential in detecting the signatures of different stages of micro-cracking in uniaxially loaded rock specimens.

Linear ultrasonic testing (LUT) is a more traditional approach for non-destructive characterization of damage in materials. LUT probes the changes in the linear acoustic wave parameters (i.e. pulse velocity, amplitude attenuation, and frequency shifts) as damage accumulates in a material.[9,10] The linear theory of ultrasonic wave propagation is based on the assumption that the pulse velocity in a medium is constant, and the density and elasticity of a medium is independent of the transmitting wave amplitude.[11] Theoretically, this assumption implies that an ultrasonic wave will not be distorted as it propagates through a medium, although its amplitude might reduce due to scattering of the acoustic energy; that is why amplitude is a more sensitive parameter for capturing signatures of damage in a medium.[12–15] Although wave attenuation is a fairly sensitive parameter for monitoring damage in a material, the changes in wave amplitude can be insignificant at low crack densities.[8,16] Also, it has been documented that the signatures of damage evaluated using LUT methods are sensitive towards the fundamental frequency of the transducers used.[17] As such, LUT methods may fail to allow for clear identification of low to moderate levels of

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damage in a material,[18–20] which is crucial for understanding the onset of damage in rocks.

Studies have shown that non-destructive techniques that have the potential to extract the elasto-dynamic non-linear response of a material have broad implications in the non-destructive evaluation of materials.[7,21,22] NLUT approaches specifically capture the non-linear response signatures, which are originated in geomaterials particularly due to the presence of inhomogeneous features, like micro-cracks, grain boundaries, joints, debonding, delamination, crystal lattice defects, etc. NLUT has the distinct advantage that it is extremely sensitive towards micro-damage as compared to LUT.[7,16,18,23–25] NLUT technique analyses the interaction of ultrasonic waves with the material, and, specifically characterizes the non-linear effects such as: (i) super and sub-harmonic generations using the Finite Amplitude technique (FA), which requires very precise acoustic measurements[11,26]; (ii) distortion in the resonance frequencies and wave attenuation with changes in excitation amplitudes using Non-linear Resonant Ultrasound Spectroscopy (NRUS)[27,28]; (iii) wave mixing interactions using Non-linear Wave Modulation Spectroscopy (NWMS)[7]; and (iv) changes in the ultrasonic times-of-flight using a frequency mixing procedure known as the Dynamic Acousto-Elastic Technique (DAET).[4,5,29–31] These techniques quantify the non-linear material behavior by using several non-linear material parameters.[4,29,32] In the same group of damage detection techniques exploiting the non-linear features of the elastic response lies the Scaling Subtraction Method (SSM).[33–37] SSM was proposed to overcome some issues affecting the more traditional NLUT damage detection techniques (for details, refer to Munoz et al.[7] and Worden et al.[24]). The most significant issue affecting NLUT methods is the low signal to noise ratio at moderate levels of damage, which, decays rapidly on moving away from the source of non-linearity, therefore requiring very precise measurements.[7,33] The benefit of SSM lies in the fact that the amplitude of the acquired non-linear signals is large enough to be easily distinguishable from the background noise. The NLUT-SSM approach has also been found to be effective in detecting presence of localized damage in composite materials,[37] without the need of localizing transducers close to the source of non-linearity (defects) in the materials. Furthermore, Antonaci et al.[36] compared the sensitivity of the LUT measurements against that of NLUT-SSM measurements for detecting damage progression in concrete, and concluded that NLUT-SSM is more effective than the traditional LUT measurements in that it can detect the distinction between initial stages of degradation and late damage propagation. As such, SSM has been previously used to identify damage in concrete and composite plates, but the effectiveness of SSM technique for detecting the presence of micro-cracking related damage in rocks has not yet been assessed.

Therefore, in the present paper, the NLUT-SSM has been tested to assess its potential for characterizing the damage processes in rocks with primary focus on the early level damage. Three rock types, Lyons sandstone (LS), granodiorite (G) and Gosford sandstone (GS) were tested for this purpose. An attempt is also made to characterize the difference in the observed non-linearity of the three rocks based on their micro-structure. The structure of the paper begins with a brief introduction on the fundamentals of non-linear ultrasonics. Next, the SSM technique has been described, followed by the testing procedure, and, then the results and conclusions are presented.

2. Fundamentals

2.1. Non-linear constitutive model

The non-linearity in a material's characteristic behavior can be understood from three perspectives[7]: (i) the continuum mechanics perspective, where the constitutive laws relating stress-strain deviate from the Hooke's law;[38] (ii) the wave equation perspective, where the non-linear parameters are directly related to the observations appearing in ultrasonics; and (iii) the micro-mechanics perspective, where physical and phenomenological models are postulated so as to explain the observed non-linearity from perspectives (i) and (ii). Since, the definitive explanation of the mechanistic origins of non-linearity is still being researched,[7] here the mathematical description of non-linearity is described based on principles of continuum mechanics.

For an elastic material, the weakly non-linear elastic theory considers a Taylor series expansion of the elastic energy function in strain powers[32,39–41]:

$$W(E) = \frac{1}{2}\lambda(\mathrm{tr}E)^2 + \mu\,\mathrm{tr}(E^2) + \frac{1}{3}A'(\mathrm{tr}E)^3 + B'(\mathrm{tr}E)\mathrm{tr}(E^2) + \frac{1}{3}C'(\mathrm{tr}E^3) \tag{1}$$

Eq. (1) is an expansion for strain energy (W) with up to third order terms in strain powers, where: λ, μ are the Lame's constants, tr denotes trace of matrice, E is Lagrangian strain tensor and A', B', C' are the Landau's third-order elastic constants.[32] The second Piola-Kirchoff stress S_{II} can be obtained in terms of Lagrangian strain E as follows:

$$S_{II} = \frac{\partial W(E)}{\partial E} \tag{2}$$

$$S_{II} = \lambda\mathrm{tr}(E)I + 2\mu E + A'(\mathrm{tr}(E))^2 I + B'(\mathrm{tr}E^2)I + 2B'(\mathrm{tr}E)E + C'E^2 \tag{3}$$

The Lagrangian strain E is computed from the displacement gradient D as per the following equations, where X denotes the Lagrangian coordinates of the material points in the reference configuration[40]:

$$D = \frac{\partial u_i(X, t)}{\partial X_j}\ (i,\ j = 1, 2, 3) \tag{4}$$

$$E = \frac{1}{2}(D + D^T + D^T D) \tag{5}$$

S_{II} can be expressed in terms of its argument D as per:

$$\begin{aligned} S_{II}(D) = {} & \tfrac{1}{2}\lambda\mathrm{tr}(D + D^T)I + \mu(D + D^T) + \tfrac{1}{2}\lambda\mathrm{tr}(DD^T)I + A'(\mathrm{tr}(D))^2 I \\ & + \mu DD^T \\ & + B'\mathrm{tr}(D)(D + D^T) + \tfrac{1}{2}B'\mathrm{tr}(D^2 + D^T D)I \\ & + \tfrac{1}{4}C'(D^2 + (D^T)^2 + D^T D + DD^T) \end{aligned} \tag{6}$$

Using Eq. (6), the stress tensor S_{II} can be decomposed into its linear and non-linear parts:

$$S_{II}(D) = S_{II}^{L}(D) + S_{II}^{NL}(D) \tag{7}$$

$$S_{II}^{L}(D) = \frac{1}{2}\lambda\mathrm{tr}\left(D + D^T\right)I + \mu\left(D + D^T\right) \tag{8}$$

$$\begin{aligned} S_{II}^{NL}(D) = {} & \tfrac{1}{2}\lambda\mathrm{tr}(DD^T)I + A'(\mathrm{tr}(D))^2 I + \mu DD^T \\ & + B'\mathrm{tr}(D)(D + D^T) + \tfrac{1}{2}B'\mathrm{tr}(D^2 + D^T D)I \\ & + \tfrac{1}{4}C'(D^2 + (D^T)^2 + D^T D + DD^T) \end{aligned} \tag{9}$$

The second Piola-Kirchoff stress S_{II} can be related with the first Piola-Kirchoff stress S_I through the deformation gradient $E^* = I + E$, by $S_I = E^* \cdot (S_{II})$, i.e.

$$S_I(E^*) = S_I^{L}(E^*) + S_I^{NL}(E^*) \tag{10}$$

For defining the geometric non-linear effects under small strains (acoustic waves) in an ideal material obeying Hooke's law, the often neglected term ($D^T D$) should be considered in the description of the material constitutive laws represented by Eq. (10).[7,41,42] However, the non-linear mechanical behavior described by this continuum-based theory of elasticity is only capable of exhibiting classical non-linear anharmonic effects arising from atomic scale forces.[43] It has been demonstrated that the non-linearity explained through this classical approach is not capable of quantitatively predicting observations in a wide class of consolidated materials like soil, cement, rocks, etc.[28,44]