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# Deposit insurance pricing under GARCH

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ABSTRACT

As homoscedasticity assumption of asset return is questionable, traditional deposit insurance pricing analysis based on the Black-Scholes model always performs poorly. This paper focuses on deposit insurance pricing under a GARCH framework. A closed-form pricing formula is derived, and an estimation method for the pricing model with market data is also presented. We apply the pricing model on a sample of 40 U.S. exchange-listed banks and the results reaffirm the importance of GARCH framework. The premium rate under the GARCH framework is always much lower than its Black-Scholes counterpart during high-risk periods.

## 1. Introduction

In a seminal paper, Merton (1977) modeled deposit insurance as a put option on the bank's assets. Since then, a stream of research has followed Merton (1977) and valuated deposit insurance within Black-Scholes option pricing framework, even though with different model specifications.[1] However, it is widely acknowledged that the constant volatility assumption of Black-Scholes framework is questionable. For example, the Black-Scholes implied volatility always contradicts with the constant volatility assumption and exhibits the "volatility smile" phenomenon as well as the term structure phenomenon. As shown in the review article by Bollerslev et al. (1992), a large number of empirical researches demonstrated that financial asset returns always exhibit many features such as volatility clustering and leverage effects, which also contradicted with the constant volatility assumption. It has become a consensus that variances of asset return change through time, and a popular choice to characterize this is GARCH models. Furthermore, several studies demonstrated that the GARCH option pricing model significantly outperforms the Black-Scholes model (e.g., Duan, 1995; Heston and Nandi, 2000; Christoffersen et al., 2008, etc.). Therefore, in this paper, we develop a pricing model for deposit insurance under GARCH framework.

Among this stream of literature, the closest research to our analysis is Duan and Yu (1999), who proposed a multi-period deposit insurance pricing model under GARCH and affirmed the importance of GARCH framework.[2] However, they could not obtain a closed-form formula, which is very essential for empirical research and practical application. Without closed-form formula, it is impossible to estimate the bank asset process and evaluate the deposit insurance premium, because the underlying asset price (i.e. the bank asset value) is unobservable. Therefore, Duan and Yu (1999)'s model under GARCH framework is less applicable in empirical research,

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[1] Some papers considered regulation policies, such as stochastic audit (Merton, 1978; Pennacchi, 1987a), capital forbearance (Ronn and Verma, 1986; Duan and Yu, 1994), and capital standard (Cooperstein et al., 1995; Pennacchi, 2005). And some other papers tried to characterize bank default risk more accurately, for example, considering interest risk (Pennacchi, 1987b; Duan et al., 1995; So and Wei, 2004), lending risk (Dermine and Lajeri, 2001), and systematic risk (Lee et al., 2015; Zhang and Shi, 2017). There were also papers taking into account some other factors affecting deposit insurance premium, such as the default risk of guaranty fund (Episcopos, 2004), and bankruptcy costs (Hwang et al., 2009).

[2] They found that the premium rate under GARCH framework is always higher than its Black-Scholes counterpart.

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especially in practices to evaluate the deposit insurance premium rate for specific banks.

In this paper, we obtain a closed-form deposit insurance pricing formula under the GARCH framework proposed by Heston and Nandi (2000). Based on the closed-form formula, we also propose a method for estimating the pricing model and evaluating deposit insurance premium rate from market data. Thus, this paper provides a rewarding tool for the empirical analysis of deposit insurance pricing practices, which is the most important contribution to the existing literature.[3] We apply the pricing model on a sample of 40 U.S. banks during 2008–2017. The results indicate that the premium rate under the GARCH framework is always much lower than its Black-Scholes counterpart during high-risk periods, because the constant volatility assumption of Black-Scholes framework overestimates the variance of asset return as well as the default risk. However, the results are inconsistent with Duan and Yu (1999), who found that the model based on Black-Scholes framework underestimates deposit insurance premium. Considering the subjective setting in their numerical analysis, our empirical results are much more reliable.[4] We also examine the effect of capital forbearance on deposit insurance pricing. And the results show that, with capital forbearance, deposit insurance premium rates increase substantially.

## 2. The deposit insurance pricing model

We consider a discrete-time economy in which time is indexed as 0, 1, 2, …, $T$. The value of the bank's asset at time $t$ is denoted by $V_t$. The face value of the deposit is denoted by $D$. As the deposit is insured, it grows at a risk-free rate. The deposit insurance with maturity $T$ is priced at time $t$.

The bank's asset value follows an asymmetric GARCH process:

$$log(V_t) = log(V_{t-1}) + r + \left(\lambda - \frac{1}{2}\right)h_t + \sqrt{h_t}\varepsilon_t \tag{1}$$

$$h_t = \omega + \alpha(\varepsilon_{t-1} - \gamma\sqrt{h_{t-1}})^2 + \beta h_{t-1} \tag{2}$$

where $h_t$ is the conditional variance of asset return, $r$ is the risk-free rate, and $\varepsilon_t$ is a standard normal disturbance. This GARCH process is slightly different from the conventional one used in Duan (1995). It is first presented in Heston and Nandi (2000), then used in Christoffersen et al. (2008) for option valuation. In both studies, the GARCH process performed well, and most importantly it provided a basis for deriving a closed-form option valuation formula.

To value the deposit insurance, the risk-neutral distribution of the bank asset at time $T$ is needed. According to Heston and Nandi (2000), the moment generating function of the logarithm of $V_T$ under the risk-neutral measure $Q$ takes the following form:

$$f(\varphi) = E_t^Q[V_T^\varphi] = V_t^\varphi exp(A(t; T, \varphi) + B(t; T, \varphi)h_{t+1}) \tag{3}$$

where

$$A(t; T, \varphi) = A(t+1; T, \varphi) + \varphi r + \omega B(t+1; T, \varphi) - \tfrac{1}{2}log(1 - 2\alpha B(t+1; T, \varphi)) \tag{4}$$

$$B(t; T, \varphi) = \varphi\left(\gamma + \lambda - \frac{1}{2}\right) - \frac{1}{2}(\gamma+\lambda)^2 + \beta B(t+1; T, \varphi) + \frac{\frac{1}{2}(\varphi - \gamma - \lambda)^2}{1 - 2\alpha B(t+1; T, \varphi)} \tag{5}$$

$A(t; T, \varphi)$ and $B(t; T, \varphi)$ can be calculated recursively from the terminal condition, $A(T; T, \varphi) = B(T; T, \varphi) = 0$.

A bank will not go bankrupt unless its asset falls below $\rho De^{r(T-t)}$, where $\rho$ represents the capital forbearance. Thus, the payoff of the deposit insurance agent at time $T$ can be described by

$$G(V_T) = \begin{cases} 0 & if\ V_T \geq \rho De^{r(T-t)}, \\ De^{r(T-t)} - V_T & otherwise. \end{cases} \tag{6}$$

The fair price of the deposit insurance should equal the present value of the expected payoff of the insurance agent under the risk-neutral measure $Q$. Following similar procedure of Heston and Nandi (2000), the deposit insurance premium rate can be derived as follows:

[3] In fact, Duan and Yu (1999) also pointed out that "Such an empirical analysis will be a fruitful undertaking in future research."

[4] Duan and Yu (1999) chose the S&P 500 series as bank asset value process to estimate the GARCH process and chose the stationary volatility as the variance of Black-Scholes model.