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Ergodicity of scalar stochastic differential equations with Hölder continuous coefficients

Luu Hoang Duc[a,b,*], Tat Dat Tran[a], Jürgen Jost[a,c]

[a] *Max-Planck-Institut für Mathematik in den Naturwissenschaften, Germany*
[b] *Institute of Mathematics, Viet Nam Academy of Science and Technology, Viet Nam*
[c] *Santa Fe Institute for the Sciences of Complexity, Santa Fe, NM 87501, USA*

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Abstract

It is well-known that for a one dimensional stochastic differential equation driven by Brownian noise, with coefficient functions satisfying the assumptions of the Yamada–Watanabe theorem (Yamada and Watanabe, 1971, [31,32]) and the Feller test for explosions (Feller, 1951, 1954), there exists a unique stationary distribution with respect to the Markov semigroup of transition probabilities. We consider systems on a restricted domain D of the phase space $\mathbb{R}$ and study the rate of convergence to the stationary distribution. Using a geometrical approach that uses the so called *free energy function* on the density function space, we prove that the density functions, which are solutions of the Fokker–Planck equation, converge to the stationary density function exponentially under the Kullback–Leibler divergence, thus also in the total variation norm. The results show that there is a relation between the Bakry–Émery curvature dimension condition and the dissipativity condition of the transformed system under the Fisher–Lamperti transformation. Several applications are discussed, including the Cox–Ingersoll–Ross model and the Ait-Sahalia model in finance and the Wright–Fisher model in population genetics.

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* Corresponding author.
E-mail addresses: duc.luu@mis.mpg.de, lhduc@math.ac.vn (L.H. Duc), trandat@mis.mpg.de (T.D. Tran), jost@mis.mpg.de (J. Jost).

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1. Introduction

Stochastic differential equations with Hölder continuous coefficients arise as models in many sciences. An instance is the Wright–Fisher diffusion model for the genetic drift of alleles with mutation [15]; in the one dimensional case (that is, when there are only two alleles present), it is of the form

$$dX_t = [\theta_1 - (\theta_1 + \theta_2)X_t]dt + \sqrt{X_t(1 - X_t)}dW_t, \quad X_t \in [0, 1],$$

where we assume that the mutation rates $\theta_1, \theta_2 > 0$. Another example is the Cox–Ingersoll–Ross [5] model for the short term interest rate

$$dX_t = k(\theta - X_t)dt + \sigma\sqrt{X_t}dW_t, \quad X_t \geq 0,$$

where $k, \theta, \sigma > 0$ satisfy $k\theta \geq \frac{\sigma^2}{2}$.

For such nonlinear systems, the diffusion coefficient is only Hölder continuous on a restricted domain [11], therefore the existence and uniqueness of a solution cannot be proved using classical arguments like contraction mappings, but one rather needs to invoke the Yamada and Watanabe theorem [31,32,24]. Moreover, the solution in general does not depend differentiably on the initial values in the state space, and thus the linearization method for studying the stability problem fails to apply here.

In this paper, we study the Markov semigroup generated by such a system; this is a strong Feller process [16,17] provided that the Feller test of explosion succeeds. In particular, assume that D is a Polish space (complete metric and separable). A probability measure μ on D is called *stationary (invariant)* with respect to the (Markov) diffusion process X_t with semigroup $(T_t)_{t\geq 0}$ on $C_0^\infty(D)$ and generator G if

$$\int_D T_t f(y)\mu(dy) = \int_D f(y)\mu(dy), \quad \forall t \geq 0, \ f \in C_0^\infty(D),$$

or equivalently (due to [19] Theorem 2.3)

$$\int_D Gf(y)\mu(dy) = 0, \quad \forall f \in C_0^\infty(D).$$

It is called *reversible* if

$$\int_D g(y)T_t f(y)\mu(dy) = \int_D f(y)T_t g(y)\mu(dy), \quad \forall t \geq 0, \ f, g \in C_0^\infty(D),$$

or equivalently

$$\int_D g(y)Gf(y)\mu(dy) = \int_D f(y)Gg(y)\mu(dy), \quad \forall f, g \in C_0^\infty(D).$$

Also, it is well-known that the density function of the transition probability satisfies the Fokker–Planck equation. The existence of stationary measures with respect to the Markov semigroup is deduced from the Krylov–Bogoliubov theorem [9]. However, when the process is reversible (with appropriate boundary conditions), we can prove directly that there exists a unique stationary measure for the Markov semigroup which can be written as a Gibbs measure $\mu_\infty(dy) = u_\infty(y)dy = \frac{e^{-\psi(y)}}{Z}dy$.

Another important issue is the rate of convergence to the stationary distribution. It is well known from the Harris theorem [23] that if there exists a Lyapunov type function, an initial distribution under the Markov semigroup will converge exponentially to the stationary

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