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Fluctuations of Omega-killed spectrally negative Lévy processes

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Abstract

In this paper we solve the exit problems for (reflected) spectrally negative Lévy processes, which are exponentially killed with a killing intensity dependent on the present state of the process and analyze respective resolvents. All identities are given in terms of new generalizations of scale functions. For the particular cases $\omega(x) = q$ and $\omega(x) = q\mathbf{1}_{(a,b)}(x)$, we obtain results for the classical exit problems and the Laplace transforms of the occupation times in a given interval, until first passage times, respectively. Our results can also be applied to find the bankruptcy probability in the so-called Omega model, where bankruptcy occurs at rate $\omega(x)$ when the Lévy surplus process is at level $x < 0$. Finally, we apply these results to obtain some exit identities for spectrally positive self-similar Markov processes. The main method throughout all the proofs relies on the classical fluctuation identities for Lévy processes, the Markov property and some basic properties of a Poisson process.

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Keywords: Lévy processes; Omega model; Occupation time; Laplace transform; Fluctuation theory; Self-similar process

1. Introduction

Exit problems for (reflected) spectrally negative Lévy processes have been the object of several studies over the last 40 years and have been used in many applied fields, such as

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mathematical finance, risk and queuing theory, biology, physics and many others. An overview can be found in [18]. The principal tools of analysis are based on the Wiener–Hopf factorization, Itô's excursion theory and the martingale theory. The aim of this paper is two-fold. Firstly, to generalize known exit identities to a ω-killed version, for which previous results are special cases, and secondly, to derive the exit identities using an alternative method, based on the properties of non-homogeneous Poisson processes and the Markov property.

Similar exit problems with functional discounting have been previously considered, in the context of optimal stopping theory, for the case of diffusion processes in Beibel and Lerche [4] and Dayanik [10] (see also references therein). This type of state-dependent killing has also been analyzed in the context of Feynman–Kac formula and option prices by Glau [13]. The results of this paper can be considered from a wider perspective when the law of the exit time, from a given set, is determined by the solution of some Dirichlet problems based on a Schrödinger-type operator with the omega potential (describing external field acting on the particle).

Let $\omega : \mathbb{R} \to \mathbb{R}_+$ be a locally bounded nonnegative measurable function and $X = \{X_t, t \geq 0\}$ be a spectrally negative Lévy process. We denote its first passage times by:

$$\tau_z^- := \inf\{t > 0 : X_t < z\} \quad \text{and} \quad \tau_c^+ := \inf\{t > 0 : X_t > c\}. \tag{1.1}$$

Throughout the paper, the law of X, such that $X_0 = x$, is denoted by $\mathbb{P}_x$ and the corresponding expectation by $\mathbb{E}_x$. We will write $\mathbb{P}$ and $\mathbb{E}$ when $x = 0$. Our main interest in this paper is deriving closed formulas for the occupation times, weighted by the ω function, considered up to some exit times. In particular, for $x \in [0, c]$ we will identify

$$\mathcal{A}(x, c) := \mathbb{E}_x\left[\exp\left(-\int_0^{\tau_c^+} \omega(X_t)\,dt\right); \tau_c^+ < \tau_0^-\right],$$

$$\mathcal{B}(x, c) := \mathbb{E}_x\left[\exp\left(-\int_0^{\tau_0^-} \omega(X_t)\,dt\right); \tau_0^- < \tau_c^+\right].$$

Applying a limiting argument to the above will produce the one-sided ω-killed exit identities for the spectrally negative Lévy process. Similar results will be derived for a reflected process at running minimum and maximum. Finally, respective resolvents are also identified.

It turns out that the identities can be characterized by two families of functions: $\{\mathcal{W}^{(\omega)}(x), x \in \mathbb{R}\}$ and $\{\mathcal{Z}^{(\omega)}(x), x \in \mathbb{R}\}$, which we will call ω-scale functions and are defined uniquely as the solutions of the following equations:

$$\mathcal{W}^{(\omega)}(x) = W(x) + \int_0^x W(x-y)\omega(y)\mathcal{W}^{(\omega)}(y)\,dy, \tag{1.2}$$

$$\mathcal{Z}^{(\omega)}(x) = 1 + \int_0^x W(x-y)\omega(y)\mathcal{Z}^{(\omega)}(y)\,dy, \tag{1.3}$$

respectively, where $W(x)$ is a classical zero scale function defined formally in (2.2).

In the case of a constant ω function, i.e. $\omega(x) = q$, we will show that the ω-scale functions reduce to the classical scale functions $(\mathcal{W}^{(\omega)}(x), \mathcal{Z}^{(\omega)}(x)) = (W^{(q)}(x), Z^{(q)}(x))$, producing the well-known exit identities for the two-sided exit problems for spectrally negative Lévy processes (see [19,18]).

Taking

$$\omega(x) = p + q\mathbf{1}_{(a,b)}(x)$$