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Manuel D. de la Iglesia, Pablo Román

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SOME BIVARIATE STOCHASTIC MODELS ARISING FROM GROUP REPRESENTATION THEORY

MANUEL D. DE LA IGLESIA AND PABLO ROMÁN

Abstract. The aim of this paper is to study some continuous-time bivariate Markov processes arising from group representation theory. The first component (level) can be either discrete (quasi-birth-and-death processes) or continuous (switching diffusion processes), while the second component (phase) will always be discrete and finite. The infinitesimal operators of these processes will be now matrix-valued (either a block tridiagonal matrix or a matrix-valued second-order differential operator). The matrix-valued spherical functions that appear in the representations of the symmetric pair $(\mathrm{SU}(2)\times\mathrm{SU}(2), \mathrm{diag}\,\mathrm{SU}(2))$ will be eigenfunctions of these infinitesimal operators, so we can perform spectral analysis and study directly some probabilistic aspects of these processes. Among the models we study there will be rational extensions of the one-server queue and Wright-Fisher models involving only mutation effects.

1. Introduction

It is very well known that many important results of one-dimensional stochastic processes can be obtained by using spectral methods. In particular, for Markov processes, many probabilistic aspects can be analyzed in terms of the (orthogonal) eigenfunctions and eigenvalues of the infinitesimal operator associated with the Markov process. In a series of papers in 1950-1960, S. Karlin and J. McGregor studied random walks and birth-and-death processes by using orthogonal polynomials (see [18]–[22]). Since the one-step transition probability matrix of the random walk or the infinitesimal operator of the birth-and-death process are tridiagonal matrices, it is possible to apply the spectral theorem to find the corresponding Borel measure associated with the process. With this measure it is easier to study the transition probabilities, the invariant measure or the behavior of the states of the process. Many other authors like M. Ismail, G. Valent, H. Dette, D. P. Maki or E. van Doorn, to mention a few, have studied this connection and other probabilistic aspects (see e.g. [4, 16, 30, 40, 41]). As for diffusion processes, it is also possible to use spectral methods, but now applied to second-order differential operators. Many authors like H. McKean, J. F. Barrett, D. G. Lampard, E. Wong or more recently D. Bakry, O. Mazet and B. Griffiths have studied this connection (see e.g. [1, 2, 3, 8, 17, 23, 32, 42]). Prominent examples are the Orstein-Uhlenbeck process, population growth models or Wright-Fisher models. For a brief account of the subject and other relations between stochastic processes and orthogonal polynomials, see [38].

A natural extension in this direction are bivariate Markov processes with discrete and finite second component. Now the state space is two-dimensional of the form $\mathcal{S}\times\{1,2,\dots,N\}$, where $\mathcal{S}\subseteq\mathbb{R}$ is either a discrete set or a continuous interval, and N is a positive integer. The first component is usually called the *level*, while the second one is called the *phase*. If $\mathcal{S}$ is discrete these processes are typically called *quasi-birth-and-death processes* (see [29, 33]), while if $\mathcal{S}$ is a continuous real interval, they are called *switching diffusion processes* (see [31, 43]). They key point to study spectral methods of these processes will be the theory of matrix-valued orthogonal

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