Available online at www.sciencedirect.com

ScienceDirect

Stochastic Processes and their Applications ▮ (▮▮▮▮) ▮▮▮–▮▮▮

stochastic processes and their applications

www.elsevier.com/locate/spa

General dynamic term structures under default risk

Claudio Fontana[a], Thorsten Schmidt[b,c,d,*]

[a] *Laboratoire de Probabilités et Modèles Aléatoires, Université Paris Diderot, avenue de France, 75205, Paris, France*
[b] *Freiburg University, Department of Mathematical Stochastics, Eckerstr. 1, 79104 Freiburg, Germany*
[c] *Freiburg Research Institute of Advanced Studies (FRIAS), Germany*
[d] *University of Strasbourg Institute for Advanced Study (USIAS), France*

Received 7 March 2016; received in revised form 1 November 2017; accepted 2 November 2017
Available online xxxx

Abstract

We consider the problem of modelling the term structure of defaultable bonds, under minimal assumptions on the default time. In particular, we do not assume the existence of a default intensity and we therefore allow for the possibility of default at predictable times. It turns out that this requires the introduction of an additional term in the forward rate approach by Heath et al. (1992). This term is driven by a random measure encoding information about those times where default can happen with positive probability. In this framework, we derive necessary and sufficient conditions for a reference probability measure to be a local martingale measure for the large financial market of credit risky bonds, also considering general recovery schemes.

Keywords: Credit risk; HJM; Arbitrage; Forward rate; Default compensator; Structural approach; Reduced-form approach; Large financial market; Recovery

1. Introduction

The study of the evolution of the term structure of bond prices in the presence of default risk typically starts from a forward rate model similar to the classical approach of Heath, Jarrow and Morton (HJM) in [30]. In this approach, bond prices are assumed to be absolutely continuous

* Corresponding author at: Freiburg University, Department of Mathematical Stochastics, Eckerstr. 1, 79104 Freiburg, Germany.
E-mail addresses: fontana@math.univ-paris-diderot.fr (C. Fontana), thorsten.schmidt@stochastik.uni-freiburg.de (T. Schmidt).

https://doi.org/10.1016/j.spa.2017.11.003
0304-4149/

with respect to the lifetime of the bond (maturity). This assumption is typically justified by the argument that, in practice, only a finite number of bonds are liquidly traded and the full term structure is obtained by interpolation, hence is smooth.

In markets with default risk, however, discontinuities are the rule rather than the exception: the seminal work of Merton [44] clearly shows such a behaviour, as do many other *structural models* (see, e.g., [3,27,28]). A default event usually occurs in correspondence of a missed payment by a corporate or sovereign entity and, in many cases, the payment dates are publicly known in advance. The missed coupon payments by Argentina on a notional of $29 billion (on July 30, 2014; see [31]) and by Greece on a notional of €1.5 billion (on June 30, 2015; see [14]) are prime examples of credit events occurring at predetermined payment dates. It is therefore natural to expect the term structure of default risky bonds to exhibit discontinuities in correspondence of such payment dates.[1]

On the other side, *reduced-form models* (see [2,12,19,34,42] for some of the first works in this direction) are less ambitious about the precise mechanism leading to default and neglect this phenomenon. Reduced-form models generally assume the existence of a *default intensity*, thus implying that the probability of the default event occurring at any predictable time vanishes. Accordingly, reduced-form HJM-type models for defaultable term structures typically postulate that, prior to default, bond prices are absolutely continuous with respect to maturity, i.e., under the assumption of zero recovery, credit risky bond prices $P(t,T)$ are described by

$$P(t,T) = \mathbb{1}_{\{\tau > t\}} \exp\left(- \int_t^T f(t,u) du \right), \tag{1.1}$$

with τ denoting the random default time and $(f(t,T))_{0 \leqslant t \leqslant T}$ an instantaneous forward rate. This approach has been studied in numerous works and up to a great level of generality, beginning with the first works [13,36,50,51] and extended in various directions in [15,16,45,49] (see [4, Chapter 13] for an overview of the relevant literature).

It turns out that, assuming absence of arbitrage, the presence of predictable times at which the default event can occur with strictly positive probability is incompatible with an absolutely continuous term structure of the form (1.1). This fact, already pointed out in 1998 in [51], has motivated more general approaches such as [3] and [26] (see Section 3.6 for an overview of the related literature). In particular, in the recent paper [26], the classical reduced-form HJM approach is extended by allowing the *default compensator* to have an absolutely continuous part, corresponding to a default intensity, as well as a discontinuous part with a finite number of jumps. The presence of jumps allows the default event to occur with strictly positive probability at the predictable jump times, which in [26] are assumed to be revealed in advance in the market. In this context, in order to exclude arbitrage possibilities, the term structure equation (1.1) has to be extended by introducing discontinuities in correspondence of those times.

In the present paper, we introduce a general framework for the modelling of defaultable term structures under minimal assumptions, going significantly beyond the intensity-based approach and generalizing the setting of [26]. More specifically, we refrain from making *any* assumption on the default time τ as well as on the default compensator, allowing in particular the default event to occur with strictly positive probability at predictable times. To the best of our knowledge, previous approaches to the modelling of defaultable term structures have always imposed some

[1] As an illustrative example, the timeline of the payment dates on Greece's debt is publicly available and daily updated at http://graphics.wsj.com/greece-debt-timeline.

Please cite this article in press as: C. Fontana, T. Schmidt, General dynamic term structures under default risk, Stochastic Processes and their Applications (2017), https://doi.org/10.1016/j.spa.2017.11.003.