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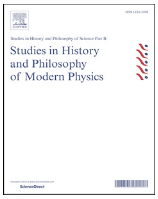

Forces on fields

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ABSTRACT

In electromagnetism, as in Newton's mechanics, action is always equal to reaction. The force from the electromagnetic field on matter is balanced by an equal and opposite force from matter on the field. We generally speak only of forces exerted by the field, not forces exerted upon the field. But, we should not be hesitant to speak of forces acting on the field. The electromagnetic field closely resembles a relativistic fluid and responds to forces in the same way. Analyzing this analogy sheds light on the inertial role played by the field's mass, the status of Maxwell's stress tensor, and the nature of the electromagnetic field.

1. Introduction

Newton's third law states that whenever one body exerts a force on a second, the second body exerts an equal and opposite force on the first. The electromagnetic field exerts forces on matter via the Lorentz force law. I will argue that matter exerts equal and opposite forces on the field.

Talk of forces on fields is generally resisted as fields seem too insubstantial to be acted upon by forces. It would be hard to understand how fields could feel forces if they had neither masses nor accelerations. Fortunately, fields have both. Fields respond to forces in much the same way that matter does.

Few authors explicitly reject the idea that matter exerts forces on the electromagnetic field. Instead, the rejection is implied by conspicuous omission. In deriving and discussing the conservation of momentum, one speaks freely of the *force* on matter but only of the *rate of change of the momentum* of the electromagnetic field (e.g., Cullwick, 1952; Griffiths, 1999, section 8.2; Rohrlich, 2007, section 4.9).

My primary goal in this article is to argue that Newton's third law holds in the special relativistic theory of electromagnetism because the force from the electromagnetic field on matter is balanced by an equal and opposite force from matter on the field. I show that the field experiences forces by giving a force law for the electromagnetic field using hydrodynamic equations which describe the flow of the field's mass (originally studied by Poincaré, 1900). In the course of this analysis I clarify the inertial role played by the field's mass—it quantifies the resistance the field itself has to being accelerated. I also point out that Maxwell's stress tensor is in fact a momentum flux density tensor, not—as its title would suggest—a stress tensor, and give the true stress tensor for the electromagnetic field. Finally, I explore the extent of the resemblance between the electromagnetic field and a relativistic fluid, asking (i) whether we can replace Maxwell's equations with fluid equations, (ii) if it is possible to understand the classical electromagnetic field as composed of photons, and (iii) how we can attribute proper mass to the field.

2. Apparent violation of the third law

If one takes charged particles to exert electromagnetic forces directly upon one another at a distance, violations of Newton's third law are easy to generate. Consider the following case (Lange, 2002, section 5.2): There are two particles of equal charge initially held in place (at rest) and separated by a distance r_1. Then, one particle is quickly moved directly towards the other as depicted in Fig. 1 so that at time t the distance between the two particles is r_2. Because there is a light-speed delay in the way charged particles interact with one another, the force that each particle feels from the other at t cannot be calculated just by looking at what's going on at t. The force on the stationary particle at t is calculated by looking at the state of the particle that moved at the time when a light-speed signal from that particle would just reach the stationary particle at t. At this earlier time, the particle was a distance r_1 from where the stationary particle is at t. The general law describing how the force on one charge depends on the state of another at an earlier

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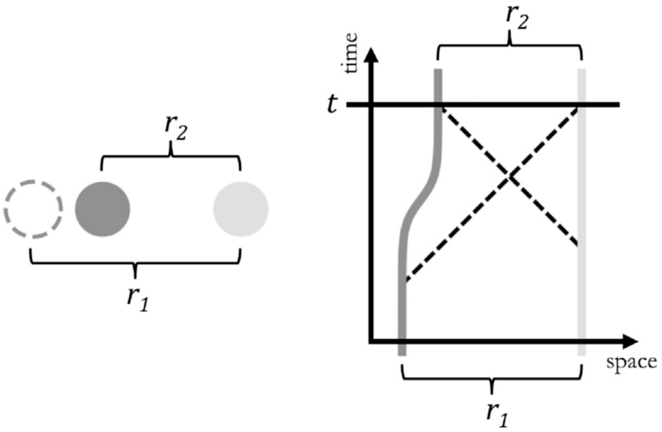

Fig. 1. The two gray lines represent spacetime trajectories of charged particles. The dotted lines indicate which point one must examine on each particle's spacetime trajectory to calculate the force on the other at *t*—taking into account the light-speed delay on interactions.

time is complex,[1] but in this simple case where both particles are at rest at the relevant times, the repulsive force that the stationary particle feels at t has magnitude $\frac{q^2}{r_1^2}$. Similarly, the force on the particle that moved is calculated by looking at the state of the stationary particle at a time when the stationary particle was at a distance r_2 from where the particle that moved is at t. The repulsive force the particle that moved feels at t has magnitude $\frac{q^2}{r_2^2}$, opposite but not equal the force on the stationary particle.

As a second example (Griffiths, 1999, section 8.2.1), imagine two particles of equal charge, both equidistant from the origin and approaching at the same speed. Particle 1 approaches along the x-axis from positive infinity and particle 2 along the y-axis. Both are guided so that they unerringly follow their straight paths at constant speed. In this case the electric forces on the two particles are equal and opposite but the magnetic forces are equal in magnitude but not opposite in direction. The magnetic force on particle 1 is in the y-direction whereas the magnetic force on 2 is in the x-direction.

According to Griffiths, we should be troubled by this violation because "…the proof of conservation of momentum rests on the cancellation of internal forces, which follows from the third law. When you tamper with the third law, you are placing the conservation of momentum in jeopardy, and there is no principle in physics more sacred than *that*." Griffiths then immediately neutralizes the threat, writing that "Momentum conservation is rescued in electrodynamics by the realization that the fields themselves carry momentum." Feynman, Leighton, & Sands (1964, sections 26-2 and 27-6) respond to apparent violations of the third law in a similar manner. They write that they will leave it to the reader to worry about whether action is equal to reaction, but point out that momentum is conserved—provided that the field momentum is included—and seem satisfied with this resolution of the puzzle.

I believe these responses capture the general attitude of physicists to the apparent violation of Newton's third law and they are correct as far as they go. However, by shifting the focus to conservation of momentum they leave the question of whether Newton's third law holds unanswered. Since conservation of momentum has been upheld and the status of Newton's third law remains uncertain, one might reasonably conclude that conservation of momentum is the deeper principle. This common attitude appears in the Wikipedia (2017) article on Newton's laws of motion: "Newton used the third law to derive the law of conservation of momentum; from a deeper perspective, however, conservation of momentum is the more fundamental idea (derived via Noether's theorem from Galilean invariance), and holds in cases where Newton's third law appears to fail, for instance when force fields as well as particles carry momentum, and in quantum mechanics." Lange (2002, pg. 163) gives a more definitive rejection of the third law as a footnote to his discussion of conservation of energy and momentum, "However, Newton's third Law ('Every action is accompanied by an equal and opposite reaction') is still violated, even if fields are real. Bodies do not exert forces on fields; bodies alone feel forces. Newton's third law was thus abandoned before relativity theory came on the scene."[2]

Another possible reaction to our quandary is to view the third law as immediately saved by the fact that momentum is conserved. If force is simply the rate of change of momentum, then the fact that the amount of momentum in the field is changing is sufficient to demonstrate that forces act on the field (presumably from matter as it is the only other actor on the scene). Because momentum is conserved, changes in momentum must cancel and thus forces must balance—Newton's third law is preserved. I think it is ultimately correct that the third law is saved by the fact that forces act on fields. However, I find this quick version of the argument unsatisfactory. One reason for dissatisfaction is that although the presence of forces on fields is suggested, a mathematical account of how forces act on fields is absent. Another problem with this quick argument is that it begs the question against someone who thinks that the conservation of momentum is a deeper principle than Newton's third law and may hold in cases where Newton's third law does not, as this argument makes obedience of the third law an immediate consequence of the conservation of momentum.

Some readers might balk at the idea that forces could act upon the electromagnetic field because they think that the field is merely a useful tool, not a real thing. If the field isn't real, it's hard to see how either Newton's third law or the conservation of momentum could hold (though some clever maneuvers have been made to save Newton's third law and conservation of momentum in field-less versions of electromagnetism; see Wheeler & Feynman, 1949; Lange, 2002, chapter 5; Lazarovici, 2017, section 4.2). Over the years, much has been said in favor of, and in opposition to, taking the electromagnetic field to be real. For my purposes here, I would like to avoid entering this debate by simply assuming a certain resolution—that the field is real—and addressing the status of the third law given this assumption. Once complete, one might take the story presented here to provide new reasons for believing the field to be real. But, I will not explicitly draw them out as this debate is not my focus.

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[1] The law giving the force that one charged particle exerts on another is calculated from the retarded Liénard-Wiechert potentials (Griffiths, 1999, chapter 10; Feynman et al., 1964, section 21-1; Lange, 2002, pg. 30; Earman, 2011, section 2). Newton's third law is violated in this sort of case because we are calculating forces directly between particles, not because of the particular choice to use retarded potentials in order to do so. If advanced potentials were used instead, a similar violation would arise if the swerve were placed in the future instead of the past. If half-retarded half-advanced potentials were used to calculate the forces between particles, either swerve would be sufficient to generate a violation.

[2] According to Frisch and Pietsch (2016, pg. 16), Ritz (1908) made a similar point while defending a version of electromagnetism without an electromagnetic field (in which charged particles act directly upon one another) and criticizing versions of the theory that include field or aether: "[Ritz] also notes that a theory presupposing an aether does not obey the equality of action and reaction, since the particle does not react back when the aether acts on a particle."

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