ARTICLE IN PRESS

Studies in History and Philosophy of Modern Physics xxx (2017) 1–9

Contents lists available at ScienceDirect

Studies in History and Philosophy of Modern Physics

journal homepage: www.elsevier.com/locate/shpsb

The asymptotic safety scenario for quantum gravity — An appraisal

Simon Friederich [a, b, *]

[a] University of Groningen, University College Groningen, Hoendiepskade 23/24, 9718 BG Groningen, The Netherlands
[b] University of Groningen, Faculty of Philosophy, Oude Boteringestraat 52, 9712 GL Groningen, The Netherlands

ARTICLE INFO

Article history:
Received 21 August 2017
Received in revised form
16 November 2017
Accepted 2 December 2017
Available online xxx

ABSTRACT

The paper has three main aims: first, to make the asymptotic safety-based approach to quantum gravity better known to the community of researchers in the history and philosophy of modern physics by outlining its motivation, core tenets, and achievements so far; second, to preliminarily elucidate the finding that, according to the asymptotic safety scenario, space-time has fractal dimension 2 at short length scales; and, third, to provide the basis for a methodological appraisal of the asymptotic safety-based approach to quantum gravity in the light of the Kuhnian criteria of theory choice.

1. Introduction

The correct quantum theory of gravity remains undiscovered and is widely regarded as among the holy grails of fundamental physics. With respect to the philosophical foundations of space-time, quantum gravitational effects are a significant unknown factor with potentially drastic impacts on space-time ontology. With respect to scientific methodology, the ongoing search for a convincing quantum theory of gravity provides illuminating case study material because underdetermination of theory by data is an acute problem in this area and not merely an artificially constructed theoretical possibility.

Our best theories of fundamental constituents of matter are quantum field theories, but our best theory of gravity—Einstein's general theory of relativity—is entirely non-quantum ("classical", for the rest of this paper). And while it is possible to regard general relativity as an effective field theory and compute the leading quantum corrections to it (Donoghue (1994)) constructing a full theory of quantum gravity that applies to phenomena associated with the Planck scale $M_P \sim 10^{19}$ *GeV* and beyond is widely regarded as an enormous challenge. With respect to such high energy scales there seems to be a profound conceptual incompatibility between quantum field theories on the one hand and general relativity on the other—an incompatibility that manifests itself in the so-called *non-renormalizability* of general relativity—which is widely believed to necessitate radically novel conceptual moves. The asymptotic safety scenario, originating from a suggestion due to Steven Weinberg (Weinberg (1979)) and first concretely worked-out by Martin Reuter (Reuter (1998)), is based on the idea that, contrary to this widespread belief, non-perturbative renormalization techniques may actually reveal that there exists, after all, a straightforward quantum field theory of gravity that is mathematically well-defined and predictive up to arbitrarily high energies.

The present paper provides a comprehensive introduction to the asymptotic safety scenario to make it better known in the foundations of physics community. Moreover, it explores the asymptotic safety scenario's ramifications for space-time foundations by focusing on its consequence that space-time at very short length scales has fractal-like properties. Finally, the paper provides a methodological appraisal of the asymptotic safety scenario in the light of Kuhn's celebrated five criteria of theory choice, highlighting various of its most interesting empirical repercussions on the way.

2. Outline of the asymptotic safety scenario for quantum gravity

2.1. General relativity is not perturbatively renormalizable

A core part of the standard procedure for turning classical field theories described in terms of an action S into quantum theories is "perturbative renormalization". Starting from the classical action S, a quantum theory can be defined by the functional integral

$$Z[J] = \int D\phi \exp\left(iS + \int \phi(x)J(x)d^4x\right), \quad (1)$$

* University of Groningen, University College Groningen, Hoendiepskade 23/24, 9718 BG Groningen, The Netherlands.
E-mail address: s.m.friederich@rug.nl.

https://doi.org/10.1016/j.shpsb.2017.12.001
1355-2198/

Please cite this article in press as: Friederich, S., The asymptotic safety scenario for quantum gravity — An appraisal, Studies in History and Philosophy of Modern Physics (2017), https://doi.org/10.1016/j.shpsb.2017.12.001

where J and ϕ are field variables and the integral over ϕ ranges over all field configurations with appropriately defined boundary conditions. (The variable "ϕ" is typically used for scalar fields, but Eq. (1) can be generalized to apply to spinor, vector, and tensor fields.) It is useful to consider not only $Z[J]$ but also the related functional $W[J]$, defined by

$$Z[J] = Z[J=0]\exp(iW[J]), \tag{2}$$

The expectation values of all observables can be obtained from $W[J]$ by taking appropriate derivatives with respect to J and evaluating for suitable field configurations. In that sense, it contains all the information about the quantum theory obtained from quantizing the classical action S.

Perturbative renormalization is the standard procedure to solve the problem that some contributions to $W[J]$ typically turn out to diverge if one tries to compute $W[J]$ using a perturbative expansion in terms of some coupling constant λ. In perturbative renormalization, these divergent contributions are in a first step regularized, i.e. kept finite, e.g. by restricting any integrals in momentum-space to momenta with absolute values below some cut-off scale Λ. In a second step, these finite contributions are absorbed into the definitions of so-called *physical* parameters, differing from the *bare* parameters in terms of which S is formulated. Only the physical, not the bare, parameters are accessible through experiments. A theory is called "perturbatively renormalizable" if only finitely many parameters must be fixed through empirical input in order to complete the regularization procedure. The theories combined in the Standard Model of elementary particle physics are perturbatively renormalizable.

Notoriously, perturbative renormalization cannot successfully be applied to our best classical theory of gravity: Einstein's general theory of relativity, defined by the Einstein-Hilbert action

$$S_{EH} = \frac{1}{16\pi G}\int d^d x\sqrt{-g}(R(g)+2\Lambda), \tag{3}$$

In this case, Newton's constant G is a prima facie natural candidate coupling constant in terms of which $W[J]$ might be expanded. But, as it turns out, it is not possible to absorb the appearing infinities into finitely many parameters derived from experiment, so the resulting theory is not perturbatively renormalizable. The theory obtained by means of this procedure can at most be used as an effective, semi-classical theory, with a limited range of validity confined to energies significantly below the Planck scale $M_P \equiv (1/G)^{1/2} \sim 10^{19}\ GeV$.

Perturbative renormalizability can be saved by adding terms that contain higher-derivatives of the metric in Eq. (3). Unfortunately, the addition of such terms leads to a quantum theory where the equations of motions are no longer *unitary* (Stelle (1977)). Unlike in ordinary quantum mechanics, total probability is not conserved in the resulting theory, which is therefore not regarded as a consistent quantum theory.

2.2. Non-perturbative renormalization

The lesson that is most widely drawn from the failure of perturbative renormalization as applied to general relativity is that any quantum theory of gravity supposed to be valid at the Planck scale or even beyond will be based on (the quantization of) new degrees of freedom and/or will abandon the framework of quantum field theory altogether.

The most famous research programmes in quantum gravity are based on this diagnosis, notably, string theory and loop quantum gravity. A more conservative response is to revisit the problem of turning general relativity into a quantum theory and consider whether this might be accomplished through some other means than perturbative renormalization. The asymptotic safety-based approach to quantum gravity rests on this idea. Reuter's pioneering work on this approach, starting with his Reuter (1998), relies on the so-called *functional renormalization group* scheme for the effective average action, which has so far remained the tool of choice for this approach.

The functional renormalization group scheme is an alternative formulation of the relation Eq. (1) between some classical action S and quantities such as $Z[J]$ and $W[J]$ which define a quantum theory in terms of S. It is most conveniently formulated in terms of the so-called *effective* action $\Gamma[\phi]$, defined as the Legendre transform of $W[J]$ through the equation

$$\Gamma[\phi] = W[J] - \int J(x)\phi(x)d^4x, \tag{4}$$

where J carries an implicit dependence on ϕ in that ϕ is defined as the solution to the equation

$$\phi(x) = \frac{\delta W[J]}{\delta J(x)}. \tag{5}$$

From the effective action $\Gamma[\phi]$ the same physical information can be derived as from $W[J]$, its Legendre transform. (Technically, $\Gamma[\phi]$ is the generating functional of *one-particle irreducible vertex functions*, from which all expectation values of physical quantities can be derived.) An advantage of using $\Gamma[\phi]$ rather than $W[J]$ is that it relates to the classical action $S[\phi]$ in a particularly simple way in that there is an in principle calculable trajectory of functionals $\Gamma_k[\phi]$ which interpolate between

$$S = \Gamma_k|_{k\mapsto\Lambda} \tag{6}$$

for some suitably chosen ultraviolet cut-off Λ and

$$\Gamma = \Gamma_k|_{k=0}. \tag{7}$$

This trajectory of functionals $\Gamma_k[\phi]$ is governed by the Wetterich exact renormalization group equation (Wetterich (1993), Morris (1994))

$$\partial_k\Gamma_k[\phi] = \frac{1}{2}Tr\left[\left(\Gamma_k^{(2)}[\phi]+\mathscr{R}_k\right)^{-1}\partial_k\mathscr{R}_k\right]. \tag{8}$$

In this equation, "*Tr*" denotes the trace operation performed over an arbitrary chosen complete set of quantum numbers (with an additional minus-sign for fermionic fields), $\Gamma_k^{(2)}[\phi]$ is the second functional derivative of $\Gamma_k[\phi]$ with respect to the field(s) ϕ, and $\mathscr{R}_k$ is a matrix-valued regulator function. Unlike the cut-off Λ considered in perturbative renormalization, which is an *ultraviolet* cut-off, it functions as an *infrared* cut-off, i.e. it suppresses contributions that are associated with momenta p whose absolute value $|p|$ is *smaller* than the renormalization scale k.

The regulator function $\mathscr{R}_k$ can be chosen freely, provided that it function as an infrared cut-off. For example, it can be chosen such that it endows contributions associated with momenta $|p|<k$ with a large artificial mass term, such that they are suppressed when performing the trace, leaving modes with $|p|\gg k$ entirely unaffected. As a consequence, by following the trajectory of Γ_k from Λ to 0, i.e. from the classical action S to the quantum effective action Γ, one successively takes into account contributions associated with

Please cite this article in press as: Friederich, S., The asymptotic safety scenario for quantum gravity — An appraisal, Studies in History and Philosophy of Modern Physics (2017), https://doi.org/10.1016/j.shpsb.2017.12.001