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Observables, disassembled

Bryan W. Roberts

Philosophy, Logic & Scientific Method, Centre for Philosophy of Natural and Social Sciences, London School of Economics & Political Science, Houghton Street, London WC2A 2AE, UK

A R T I C L E I N F O

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A B S T R A C T

How should we characterise the observable aspects of quantum theory? This paper argues that philosophers and physicists should jettison a standard dogma: that observables must be represented by self-adjoint or Hermitian operators. Four classes of non-standard observables are identified: normal operators, symmetric operators, real-spectrum operators, and none of these. The philosophical and physical implications of each are explored.

1. Introduction

There is a disconnect between standard accounts of mathematical representation and standard accounts of physical observables. From the perspective of the philosophy of representation, we enjoy extraordinary freedom in choosing what mathematical objects can represent things. In contrast, most well-developed accounts of observables insist on restricting to a tiny corner of mathematics involving real numbers. Nowhere is this dogma more stark than in quantum mechanics, where observables are generally associated with the real-number eigenvalues of self-adjoint operators. My aim in this paper is to show how this restriction on quantum observables can be given up, and to identify the important new classes of observables that arise as a consequence.

The restriction to real numbers is sometimes motivated by appeal to an old worry about complex numbers, which should be immediately dispelled. Consider a bead that is constrained to move on a ring. We could represent its position using pairs (r, θ) of real numbers, or using the complex circle $Re^{i\theta} \in \mathbb{C}$ with $R \in \mathbb{R}^+$ and $\theta \in [0, 2\pi)$. Of course, there was once considerable scepticism about the status of complex numbers, which led to the use of the word 'imaginary' in describing them.[1] But such misgivings should not trouble us today: the complex numbers can be constructed axiomatically in just the same sense as the real numbers. So, it is difficult to see a sense in which the two representations are not equally adequate. Viewing the real and the complex circles as embedded in $\mathbb{C}^2$, we even find the two are related by a rigid rotation, shown in Fig. 1.

Nevertheless, textbook discussions of quantum theory almost always insist that observables must involve real numbers and self-adjoint operators, as in Sakurai's classic treatment: "[w]e expect on physical grounds that an observable has real eigenvalues …. That is why we talk about Hermitian observables in quantum mechanics" (Sakurai, 1994, §1.3). Similarly, Griffiths writes, "the expectation value of an observable quantity has got to be a real number (after all, it corresponds to actual measurements in the laboratory, using rulers and clocks and meters)" (Griffiths, 1995, §3.3). And Weinberg writes, "[w]e can now see why it is important for all operators representing observable quantities to be Hermitian. … Hermitian operators have real expectation values" (Weinberg, 2013, p.24). Even when one encounters quantum field operators that are not self-adjoint, such as the free Klein-Gordon field, this is quickly explained away as equivalent to a commuting pair of operators that are self-adjoint.

The philosophy of quantum mechanics has largely followed the textbooks. For example, Hughes writes that self-adjoint operators "represent physical quantities, and their eigenvalues will be the possible values of those quantities; clearly it befits a measurable quantity that its possible values should be real" (Hughes, 1992, p.33). Similarly, Albert's book on the philosophy of quantum mechanics sets out what he calls 'principle (B)', that measurable properties are to be represented by linear operators, and then

E-mail address: b.w.roberts@lse.ac.uk.

[1] Cardano derived complex solutions to the equation $x^2 - 10x + 30 = 0$ in his 1545 *Ars Magna*, but concluded, "So progresses arithmetic subtlety the end of which, as is said, is as refined as it is useless" (Cardano, 1968, §37). Over 200 years later Euler took a similar view: "they are usually called *imaginary quantities*, because they exist merely in the imagination", although he argued that "nothing prevents us from making use of these imaginary numbers, and employing them in calculation" (Euler, 1822, p.43).

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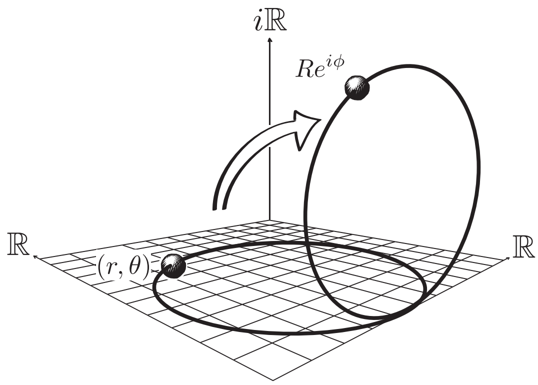

Fig. 1. Real and complex descriptions of particle position related by a rotation in $\mathbb{C}^2$.

states, “it's clear from principle (B) (since, of course, the values of physically measurable quantities are always real numbers) that the operators associated with measurable properties must necessarily be Hermitian operators” (Albert, 1992, p.40). Similar remarks are found in many other places in physics and philosophy.

The thesis of this paper is that the orthodoxy should be given up: there are many physically and philosophically interesting ways to have a non-self-adjoint observable. In particular, the self-adjointness property may be broken down into three ‘component’ properties: being normal, being symmetric, and having a real spectrum, each defined precisely below. Observables can be represented by non-self-adjoint operators that have any one of these properties while giving up the other two, or that give up all three.

The unorthodox observables that I will advocate here have been discussed before. Indeed, we will see that each has been advocated by prominent physicists, and that two in particular are associated with active research programmes: symmetric operators amount to a positive operator-valued measure or ‘POVM’ approach to quantum observables, while the real-spectrum condition forms the basis for so-called ‘*PT*-symmetric’ quantum theory. The ‘normal operators’ approach is not as well-understood, and so I will develop it beyond existing discussions. However, my aim here is not to introduce new physics. Rather, I would like to reduce some of the confusion that philosophical and textbook treatments of observables appear to have introduced. I also aim to clarify the connections between these unorthodox research programmes. It is striking that few physicists advocating one of the non-standard approaches appear to recognise any of the others. I hope this discussion might help improve their mutual recognition, by identifying the role that each plays in the philosophical foundations of observables.

The plan of the paper is as follows. The second section will introduce the dogma of self-adjoint operators, and then propose a way to classify the possible non-self-adjoint observables. The third section considers non-self-adjoint operators that are normal. Here I argue that existing proposals in favour of normal operators must be restricted using the concept of what I call a ‘sharp set’. The fourth section explores the physics of non-normal operators. First I consider non-normal operators that are symmetric but do not have a real spectrum; these turn out to amount to a ‘POVM’ approach to observables, and also allow for the introduction of ‘time observables’. Next, I consider operators that have a real spectrum but are not symmetric; these include *PT*-symmetric observables. Finally, I consider operators that do not have any of these three properties: they are not normal, do not have a real spectrum, and are not symmetric. The fifth section is the conclusion.

2. Self-adjointness disassembled

2.1. The history of self-adjointness

How did we come to require self-adjoint observables? It began when Heisenberg arrived in Göttingen in June of 1925 with a draft of his celebrated paper on non-commutative mechanics. Max Born famously recognised, upon seeing this draft, that the theory could be represented in terms of matrices. Soon, Born and Jordan (1925) had formulated the observables of quantum mechanics as self-adjoint or ‘Hermitian’ operators.[2] In a letter to Jordan in September of that year, Heisenberg wrote, “Now the learned Göttingen mathematicians talk so much about Hermitian matrices, but I do not even know what a matrix is”.[3] As Heisenberg's letter reveals, matrices were far from common tools among physicists at the time, let alone Hermitian ones, despite the latter having been introduced by Hermite (1855) seventy years earlier.

Physically significant non-Hermitian matrices appeared the following May, when London (1926) derived the non-Hermitian raising and lowering operators for the harmonic oscillator. By December of 1926, Jordan (1927a) was actually toying with the idea of treating non-Hermitian operators as observables. Remarkably, Jordan's formalism allowed one to assign complex expectation values to such non-Hermitian operators, as Duncan and Janssen (2013, §2.4) have shown. But in April of 1927, Hilbert, von Neumann and Nordheim had identified self-adjoint operators as appropriate for ensuring that the values of energy are always positive numbers.[4] By the time Jordan (1927b) submitted a follow-up paper in June, he had given up on the idea of non-Hermitian observables in favour of the new dogma.[5]

Like many aspects of quantum theory as we know it, self-adjointness was consolidated at the September 1927 Solvay conference, where Born and Heisenberg's report argued that, “the analogy with classical [Fourier] theory leads further to allowing as representatives of real quantities only matrices that are Hermitian” (Born & Heisenberg, 2009, p.327). Their idea is a familiar one: it is often convenient to use a complex unit $e^{i\theta} = \cos\theta + i\sin\theta$ to represent a harmonic phenomenon like a classical wave, on the understanding that a physical wavecrest is described by just the real part, $\mathrm{Re}(e^{i\theta}) = \cos\theta$.

The dogma soon became encoded in the influential textbooks of the field, including Dirac's famous *Principles of Quantum Mechanics*. In the 1930 first edition, Dirac actually used the term ‘observables’ to refer to all linear operators. But he quickly revised this language by the second edition of 1935, writing, “it is preferable to restrict the word ‘observable’ to refer to real functions of dynamical variables and to introduce a corresponding restriction on the linear operators that represent observables” (Dirac, 1935, p.29). The ‘corresponding restriction’ was that observables be self-adjoint (for a

[2] Charmingly, their collaboration apparently began by chance, on a train to Hannover soon after Born met Heisenberg in 1925. Born recalls confiding to a colleague on the train that he had formulated Heisenberg's equations of motion using matrix theory, but was stuck trying to derive the energy from this. Jordan, who was sitting opposite and overheard the conversation, said, “Professor, I know about matrices, can I help you?” Born suggested they give it a try, and a historic collaboration ensued (from an interview with Born by Ewald, 1960).

[3] Quoted from Jammer (1996, p.207) The impressive list of ‘learned mathematicians’ at Göttingen when Heisenberg arrived in 1925 includes Paul Bernays, Max Born, Richard Courant, David Hilbert, Pascual Jordan, Emmy Noether, Lothar Nordheim, B.L. Van der Waerden, and Hermann Weyl.

[4] (Hilbert, von Neumann, & Nordheim, 1928). As Janssen and Duncan point out, this article was submitted in April 1927, but “for whatever reason” not published until 1928 (Duncan & Janssen, 2013, §3, p.221).

[5] See Duncan and Janssen (2009, 2013) for a fascinating exposition of this episode in the development of quantum mechanics.

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