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Boundedness for solutions of equations of the Chern-Simons-Higgs type

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Abstract This paper is concerned with a priori estimate of an equation of the Chern-Simons-Higgs type. We study the uniform bound of classical solutions u of the semilinear equation and integrable solutions u of the fractional order equation. We prove $|u| \leq 1$ in R^n, which comes into play in studying the quantization effects of those equations.
Keywords: Chern-Simons-Higgs equation, fractional Laplacian, uniform bound
MSC2010: 35Q56, 35R11, 35J61, 35B45

1 Introduction

In 1994, Brezis, Merle and Riviere [1] studied the quantization effects of the following equation

$$-\triangle u = (1-|u|^2)u \quad in\ R^2.$$

It is the Euler-Lagrange equation of the Ginzburg-Landau energy

$$E_{GL}(u) = \frac{1}{2}\|\nabla u\|^2_{L^2(R^2)} + \frac{1}{4}\|1-|u|^2\|^2_{L^2(R^2)}.$$

Here $u : R^2 \to R^2$ is a vector value function. In particular, they proved $|u| \leq 1$ in R^2 (see also [6] and [12]). Afterwards, the same conclusion was obtained in [11] for the following equation

$$-\triangle u = |u|^2(1-|u|^2)u - \frac{1}{2}(1-|u|^2)^2 u \quad in\ R^n \tag{1.1}$$

under the assumption

$$\nabla u \in L^2(R^n), \quad \text{or} \quad |u|^2 - 1 \to 0 \text{ when } |x| \to \infty \tag{1.2}$$

in the case of $n = 2$. Such an equation appears in the study of the variant of the Ginzburg-Landau energy (cf. [9], [10] and [11]) and the Chern-Simons-Higgs (CSH) energy (cf. [7] and [8])

$$E_{CSH}(u) = \frac{1}{2}\|\nabla u\|^2_{L^2(R^2)} + \frac{1}{4}\|u(1-|u|^2)\|^2_{L^2(R^2)}.$$

It is also related to the study of the Maxwell-Chern-Simons model (cf. [3], [4], [5]). In this paper, we will prove

$$|u| \leq 1, \quad in\ R^n\ (n \geq 2) \tag{1.3}$$