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ON WELL-POSEDNESS OF A DISPERSIVE SYSTEM OF THE WHITHAM–BOUSSINESQ TYPE

EVGUENI DINVAY

ABSTRACT. The initial-value problem for a particular bidirectional Whitham system modelling surface water waves is under consideration. This system was recently introduced in [7]. It is numerically shown to be stable and a good approximation to the incompressible Euler equations. Here we prove local in time well-posedness. Our proof relies on an energy method and a compactness argument. In addition some numerical experiments, supporting the validity of the system as an asymptotic model for water waves, are carried out.

1. INTRODUCTION

We regard the Cauchy problem for the system that in non-dimensional variables has the form

$$\eta_t = -v_x - i \tanh D(\eta v), \quad (1.1)$$

$$v_t = -i \tanh D \eta - i \tanh D v^2 / 2 \quad (1.2)$$

where $D = -i\partial_x$ and so $\tanh D$ is a bounded self-adjoint operator in $L_2(\mathbb{R})$. The system models the two-dimensional water wave problem for an inviscid incompressible flow. As usual η denotes the surface elevation. Its dual variable v roughly speaking has the meaning of the surface fluid velocity. In the Boussinesq regime it coincides with the horizontal fluid velocity at the surface.

Equations (1.1)-(1.2) appeared in literature recently as an alternative to other linearly fully dispersive models able to describe two-wave propagation [7]. Those models capture many interesting features of the full water wave problem and are in a good agreement with experiments [4]. As to well-posedness, the existing results for them are not satisfactory. For example, the system regarded in [8] is locally well posed if only an additional non-physical condition $\eta \geq C > 0$ is imposed. This system is probably ill-posed for large data if one removes the assumption $\eta > 0$. An heuristic argument is given in [12]. This is not a problem for System (1.1)-(1.2).

Another important property of System (1.1)-(1.2) is its Hamiltonian structure. Indeed, regarding the functional

$$\mathcal{H}(\eta, v) = \frac{1}{2} \int_{\mathbb{R}} \left(\eta^2 + v \frac{D}{\tanh D} v + \eta v^2 \right) dx$$

Equations (1.1)-(1.2) can be rewritten in the form

$$\partial_t(\eta, v)^T = J \nabla \mathcal{H}(\eta, v)$$

with the skew-adjoint matrix

$$J = \begin{pmatrix} 0 & -i \tanh D \\ -i \tanh D & 0 \end{pmatrix}.$$

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