# Accepted Manuscript


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PII: S0893-9659(18)30282-9
DOI: https://doi.org/10.1016/j.aml.2018.08.005
Reference: AML 5616

To appear in: *Applied Mathematics Letters*

Received date : 20 May 2018
Revised date : 8 August 2018
Accepted date : 8 August 2018

Please cite this article as: E. Dinvay, On well-posedness of a dispersive system of the Whitham–Boussinesq type, Appl. Math. Lett. (2018), https://doi.org/10.1016/j.aml.2018.08.005

# ON WELL-POSEDNESS OF A DISPERSIVE SYSTEM OF THE WHITHAM–BOUSSINESQ TYPE

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Abstract. The initial-value problem for a particular bidirectional Whitham system modelling surface water waves is under consideration. This system was recently introduced in [7]. It is numerically shown to be stable and a good approximation to the incompressible Euler equations. Here we prove local in time well-posedness. Our proof relies on an energy method and a compactness argument. In addition some numerical experiments, supporting the validity of the system as an asymptotic model for water waves, are carried out.

## 1. Introduction

We regard the Cauchy problem for the system that in non-dimensional variables has the form

$$\eta_t = -v_x - i \tanh D(\eta v), \tag{1.1}$$

$$v_t = -i \tanh D\eta - i \tanh D v^2/2 \tag{1.2}$$

where $D = -i\partial_x$ and so $\tanh D$ is a bounded self-adjoint operator in $L_2(\mathbb{R})$. The system models the two-dimensional water wave problem for an inviscid incompressible flow. As usual $\eta$ denotes the surface elevation. Its dual variable $v$ roughly speaking has the meaning of the surface fluid velocity. In the Boussinesq regime it coincides with the horizontal fluid velocity at the surface.

Equations (1.1)-(1.2) appeared in literature recently as an alternative to other linearly fully dispersive models able to describe two-wave propagation [7]. Those models capture many interesting features of the full water wave problem and are in a good agreement with experiments [4]. As to well-posedness, the existing results for them are not satisfactory. For example, the system regarded in [8] is locally well posed if only an additional non-physical condition $\eta \geqslant C > 0$ is imposed. This system is probably ill-posed for large data if one removes the assumption $\eta > 0$. An heuristic argument is given in [12]. This is not a problem for System (1.1)-(1.2).

Another important property of System (1.1)-(1.2) is its Hamiltonian structure. Indeed, regarding the functional

$$\mathcal{H}(\eta, v) = \frac{1}{2}\int_{\mathbb{R}} \left( \eta^2 + v\frac{D}{\tanh D}v + \eta v^2 \right) dx$$

Equations (1.1)-(1.2) can be rewritten in the form

$$\partial_t(\eta, v)^T = J\nabla\mathcal{H}(\eta, v)$$

with the skew-adjoint matrix

$$J = \begin{pmatrix} 0 & -i\tanh D \\ -i\tanh D & 0 \end{pmatrix}.$$

*Date*: August 8, 2018.