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The Stability of Curved Fronts in a Periodic Shear Flow

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Abstract

Recently, El Smaily, Hamel and Huang [4] proved the existence of curved fronts in a periodic shear flow, which satisfy some "conical" conditions at infinity in the whole plane $\mathbb{R}^2$. In this paper, we study the stability of these curved fronts. In fact, we prove that all the curved fronts are exponentially stable provided that the initial perturbations decay exponentially with an appropriate rate in the lower cone.

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Keywords: Curved fronts; Stability; Reaction-advection-diffusion equation.

1 Introduction

In this paper, we consider the stability of curved fronts for the following reaction-advection-diffusion equation

$$\frac{\partial u}{\partial t} = \Delta u + q(x)\frac{\partial u}{\partial y} + f(u), \quad 0 < u(x,y,t) < 1, \quad \forall t \in \mathbb{R},\ (x,y) \in \mathbb{R}^2, \tag{1.1}$$

where the advection coefficient $q(x)$ belongs to $C^{1,\delta}(\mathbb{R})$ for some $\delta > 0$, and satisfies

$$\forall x \in \mathbb{R}, \quad q(x+L) = q(x), \quad \text{and} \quad \int_0^L q(x)dx = 0 \tag{1.2}$$

for some positive constant L. The nonlinearity f is assumed to satisfy the following conditions

$$\begin{cases} f \text{ is defined on } \mathbb{R}, \ \text{Lipschitz continuous, and } \ f \equiv 0 \text{ in } \mathbb{R} \setminus (0,1), \\ f \text{ is a concave function of class } C^{1,\delta} \text{ in } [0,1], \\ f'(0) > 0,\ f'(1) < 0, \text{ and } f(s) > 0 \text{ for all } s \in (0,1). \end{cases} \tag{1.3}$$

A typical example of such a function f is the quadratic nonlinearity $f(u) = u(1-u)$, which was initially considered by Fisher [6], Kolmogorov, Petrovsky and Piskunov [9]. The equation (1.1) arises in various biological models, such as population dynamics and gene developments where u stands for the relative concentration of some substance (see Aronson and Weinberger [1], Fife [5] and Murray [10] for details). In combustion, the equation (1.1) can be used to describe the models of flames in a shear flow, like in simplified Bunsen flames models with a perforated burner, and u stands for the normalized temperature.

By a travelling wave for the equation (1.1), we understand a solution $u(x,y,t)$ such that

$$u(x,y,t) = \phi(x, y + c_0 t), \tag{1.4}$$

for all $(x,y,t) \in \mathbb{R}^2 \times \mathbb{R}$, and for some positive constant c_0 which denotes the speed of propagation in the vertical direction $-y$. Then, the function ϕ satisfies the following elliptic equation

$$\Delta\phi + (q(x) - c_0)\partial_y \phi + f(\phi) = 0, \quad \forall (x,y) \in \mathbb{R}^2, \tag{1.5}$$

where the notation $\partial_y \phi$ means the partial derivative of the function ϕ with respect to the variable y.

We assume that the solutions ϕ of Eq. (1.5) are normalized so that $0 \le \phi \le 1$. In this paper, we are interested in the solutions ϕ which satisfy the following "conical" conditions at infinity

$$\begin{cases} \lim\limits_{l \to -\infty} \sup\limits_{(x,y) \in C^-_{\alpha,\beta,l}} \phi(x,y) = 0, \\ \lim\limits_{l \to +\infty} \inf\limits_{(x,y) \in C^+_{\alpha,\beta,l}} \phi(x,y) = 1, \end{cases} \tag{1.6}$$

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