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On newforms of half-integral weight and Jacobi forms

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ABSTRACT

Let N be an odd and square-free integer. Let α be a positive integer with $\alpha = 2$ or $\alpha = 5$. Let χ modulo N be a Dirichlet character and let $\chi_0 = \left(\frac{4\chi(-1)}{\cdot}\right)\chi$. Let

(a) χ and χ^2 are primitive characters mod N, if $\alpha = 2$;
(b) χ is the principal character if $\alpha = 5$.

In this paper, we set up the theory of newforms for the space of cusp forms of weight $k+1/2$ for $\Gamma_0(2^\alpha N)$ with character χ_0. Moreover, we prove that the space of newforms of weight $k + 1/2$ for $\Gamma_0(32N)$ is trivial. Also, we set up the theory of newforms for the space of Jacobi cusp forms and skew-holomorphic Jacobi cusp forms.

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1. Introduction

Let k, α be positive integers with $k \geq 2$, $\alpha \geq 2$ and let N be an odd and square-free integer. Let χ be a Dirichlet character modulo $2^\alpha N$ with $\epsilon := \chi(-1)$ and let $\chi_0 := \left(\frac{4\epsilon}{\cdot}\right)\chi$. Let $S_{k+1/2}(2^\alpha N, \chi_0)$ be the space of cusp forms of weight $k+1/2$ for $\Gamma_0(2^\alpha N)$ with character χ_0. Let $S_{2k}(2^{\alpha-2}N, \chi^2))$ be the space of cusp forms of weight $2k$, level $2^{\alpha-2}N$ with character χ^2. If χ is the principal character, then we denote $S_{k+1/2}(2^\alpha N, \chi_0)$ by $S_{k+1/2}(2^\alpha N)$ and $S_{2k}(2^{\alpha-2}N, \chi^2)$ by $S_{2k}(2^{\alpha-2}N)$.

By the works of [4] and [12], the theory of newforms for the plus space $S^+_{k+1/2}(4N, \chi_0)$ and for the full space are known, when N is odd and square-free integer and $\chi^2 = 1$.

Also, in [9], the theory of newforms have been studied for the spaces $S_{k+1/2}(8N)$ and $S_{k+1/2}(16N, \chi_0)$ with χ is a real character. Moreover, it was proved in [9] that $S^{+,new}_{k+1/2}(8N) = S^{new}_{k+1/2}(8N)$ and $S^{new}_{k+1/2}(16N) = \{0\}$, but the space $S^{new}_{k+1/2}(16N, \left(\frac{8}{\cdot}\right))$ is isomorphic to the space $S^{new}_{2k}(8N)$ under certain linear combinations of Shimura lifts. It is natural to look into the next case (with respect to the even power), where this phenomenon occurs. In this paper, we first study the theory of newforms for the space $S_{k+1/2}(32N)$ and derive that

$$S^{new}_{k+1/2}(32N) = \{0\} = S^{new}_{k+1/2}\left(32N, \left(\frac{8}{\cdot}\right)\right).$$

Then, we consider the plus space $S^+_{k+1/2}(4N, \chi_0)$ and the space $S_{k+1/2}(4N, \chi_0)$, where χ and χ^2 are primitive characters modulo N (N odd and square-free) and set up the theory of newforms. In order to derive the results, we use the dimension formulas for the respective spaces of integral and half-integral weight cusp forms and the existence of non-zero Shintani lifts on each of the normalised Hecke eigenforms of integral weight. We briefly mention these facts as follows.

(1) If χ is a Dirichlet character modulo N with N is an odd and square-free integer, we have the dimension equality. In this direction, we refer to Niwa [16], Kohnen [4], Ueda [22] and Kojima [6].

$$\text{(I) } \dim S_{k+1/2}(4N, \chi_0) = \dim S_{2k}(2N, \chi^2),$$
$$\text{(II) } \dim S^+_{k+1/2}(4N, \chi_0) = \dim S_{2k}(N, \chi^2).$$

If $\alpha \geq 5$, then, by using the dimension formulas given in [13] and [17], we derive the following dimension relation in section 3.

$$\text{(III) } \dim S_{k+1/2}(2^\alpha N) = 2 \dim S_{2k}(2^{\alpha-2}N).$$

(2) If χ is a primitive Dirichlet character modulo N, then from the explicit Waldspurger formula along with the existence of infinitely many odd fundamental discriminants $D \equiv 1 \pmod 4, (D, N) = 1$, with the non-vanishing of the special values

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