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On incompressible oblique impinging jet flows ☆

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Abstract

The object of this work is to investigate the fluid mechanics of oblique impinging jet flows and to this end some existence and nonexistence results are initiated. First, we established the existence of incompressible oblique impinging jet plane flows with two asymptotic directions. More precisely, given a two-dimensional semi-infinitely long nozzle and a wall behind the nozzle, for any given mass flux in the inlet of the nozzle, then there exists a smooth incompressible oblique impinging jet flow with two asymptotic directions. The impinging jet develops two free streamlines, which initiate smoothly at the endpoints of the semi-infinitely long nozzle, and the speed on free streamlines remains a constant, which can be determined by the impinging jet flow itself. The asymptotic behaviors of the oblique impinging jet flows at the far fields, the position of the stagnation points, convexity of the free boundaries and other properties are also considered. On another side, it is showed in this paper that there does not exist an oblique impinging jet flow with one asymptotic direction generally.

MSC: 76B10; 76B03; 35Q31; 35J25

Keywords: Existence and nonexistence; Impinging jet; Incompressible inviscid flow; Free streamlines

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Contents

1. Introduction and definition of the oblique impinging jet problem

The fluid mechanics of the flow generated by a Vertical/Short Takeoff and Landing (V/STOL) aircraft in ground effect is very complex. To optimize the design of these aircrafts and to predict their performance in ground effect requires a good understanding of the impinging jet-induced flow field. The mathematical theory of impinging jets has already attracted the interest of mathematicians and engineers, involved with the design of V/STOL aircraft, modern jet power plants and related machinery. On the other side, impinging jets are also important in Chemical Vapor Deposition (CVD) process, as well as being the basic flow in which strained planar flames are stability. At present, experimental studies are the main avenue followed to understand V/STOL flows. The principle focus of this paper is to investigate the well-posedness of the impinging jets based on the steady incompressible Euler equations in two dimensions.

Investigation of the mathematical topic on well-posedness connected with the free streamlines from jets can be traced back to A. Weinstein in [30] for a continuity approach. He established the first general existence theorem of a jet flow from a given polygonal channel. Later, the result was extended to a wider class of concave symmetric nozzles without limitation by R. Finn in [21]. Leray–Schauder fixed point method has been another powerful source to the existence theory, which is basically extension of continuity method to some functional spaces to obtain the several results to free boundary problem, such as infinite cavity problem solved by Leray. A fundamental advance, greatly broadening the well-posedness theory, was provided by H. Alt, L. Caffarelli and A. Friedman with their application of the variational method. This approach, together with the properties of the free boundaries developed in [1,7], has contributed a variety of the significant results to the jet flow problems, such as axially symmetric jet flow in [4], asymmetric jet flow in [2], jets with gravity in [3], jets with two fluids in [5,6]. Recently, for the impinging jet problem, the authors established the well-posedness results on the axillary symmetric impinging jet flow in [12], incompressible symmetric impinging jet flow with constant vorticity in [14] and com-

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