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Averaging principle for one dimensional stochastic Burgers equation

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Abstract

In this paper, we consider the averaging principle for one dimensional stochastic Burgers equation with slow and fast time-scales. Under some suitable conditions, we show that the slow component strongly converges to the solution of the corresponding averaged equation. Meanwhile, when there is no noise in the slow component equation, we also prove that the slow component weakly converges to the solution of the corresponding averaged equation with the order of convergence $1 - r$, for any $0 < r < 1$.

MSC: primary 34D08, 34D25 ; secondary 60H20

Keywords: Stochastic Burgers' equation; Averaging principle; Ergodicity; Invariant measure; Strong convergence; Weak convergence

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1. Introduction

Many multiscale problems arise from material sciences, chemistry, fluids dynamics, biology, ecology, climate dynamics and other application areas, see, e.g., [1,12,19,23,27,29,36] and references therein. E and Engquist [12] pointed out "Problems in these areas are often multiphysics in nature; namely, the processes at different scales are governed by physical laws of different character: for example, quantum mechanics at one scale and classical mechanics at another." For instance, dynamics of chemical reaction networks often take place on notably different times scales, from the order of nanoseconds (10^{-9} s) to the order of several days, the use of two-time or multi-time scales is common. Another example with multiple time scales is that of protein folding. While the time scale for the vibration of the covalent bonds is on the order of femtoseconds (10^{-15} s), folding time for the proteins may very well be on the order of seconds.

Many two-time scale/slow-fast systems can be formally written as

$$\begin{cases} dX_t^\varepsilon = b_1(X_t^\varepsilon, Y_t^\varepsilon)dt + \sigma_1(X_t^\varepsilon, Y_t^\varepsilon)dW_t^1, \quad X_0^\varepsilon = x \\ dY_t^\varepsilon = \frac{1}{\varepsilon} b_2(X_t^\varepsilon, Y_t^\varepsilon)dt + \frac{1}{\sqrt{\varepsilon}}\sigma_2(X_t^\varepsilon, Y_t^\varepsilon)dW_t^2, \quad Y_0^\varepsilon = y, \end{cases} \tag{1.1}$$

where W_t^1, W_t^2 are independent Wiener processes, and the small parameter ε quantifies the ratio of the X^ε and Y^ε time scales. For many practical problems, it is of interest to study the behavior of the system (1.1) for $\varepsilon << 1$, and how dynamics of this system depends on ε as $\varepsilon \to 0$. However, since $\varepsilon << 1$, it is often very difficult to directly calculate X^ε, and systems of this type are problematic for computer simulations. The averaging principle can be applied to solve these problems of this type. Roughly speaking, if the dynamics for Y^ε with $X^\varepsilon = x$ fixed has an invariant probability measure $\mu^x(dy)$ and the following integrals exist

$$\bar{b}_1(x) := \int b_1(x,y)\mu^x(dy), \quad \bar{\sigma}_1(x) := \int \sigma_1(x,y)\mu^x(dy),$$

then under appropriate assumptions on all coefficients in the system (1.1), the effective dynamics for X^ε in the limit of $\varepsilon \to 0$ is a stochastic differential equation:

$$d\bar{X}_t = \bar{b}_1(\bar{X}_t)dt + \bar{\sigma}_1(\bar{X}_t)dW_t^1, \quad \bar{X}_0 = x.$$

The theory of averaging principle has a long history and rich results. Bogoliubov and Mitropolsky [2] first studied the averaging principle for the deterministic systems. Later on, the theory of averaging principle for stochastic differential equations was firstly established by Khasminskii [21]. Since then, averaging principle for stochastic reaction–diffusion systems has become an active research area which attracted much attention. For example, Cerrai and Freidlin [5] proved the averaging principle for a general class of stochastic reaction–diffusion systems, which extended the classical Khasminskii-type averaging principle for finite dimensional systems to infinite dimensional systems. Recently, based on the averaging principle, the fast flow asymptotics for a stochastic reaction–diffusion–advection equation are obtained by Cerrai and Freidlin [6]. We refer to [3,4,13,15–18,22,24,33–35] and references therein for more interesting results on this topic.

To the best of our knowledge, there are rarely studies to deal with highly nonlinear term on this topic. In this paper, we are interested in studying the averaging principle for one dimensional

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