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Equilibrium states for random lattice models of hyperbolic type

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Abstract

We study the structural stability of random coupled map lattice models of hyperbolic type under certain metrics. Furthermore, by describing the thermodynamic formalism of the underlying random spin lattice system, we prove the existence of equilibrium states for equi-Hölder continuous random functions on lattice models under the conditions of random weak interaction and translation invariance and present some partial results on the uniqueness of equilibrium states.

MSC: 37H15; 37D20

Keywords: Equilibrium states; Random hyperbolic systems

1. Introduction

Originating from statistical mechanics, the concepts of Gibbs and equilibrium states play an important role in smooth ergodic theory in the framework of the thermodynamic formalism. Via Markov partitions, dynamical systems of hyperbolic type can be represented by thermodynamic formalism of sub-shifts of finite type, and the existence and uniqueness of equilibrium states corresponding to Hölder continuous functions are established by this classical and power-

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ful approach [5,30,31]. Moreover, it provides an abundance of equilibrium states (such as SRB measures) which have strong mixing properties and coincide, in this case, with Gibbs states. The equilibrium states theory for random transformations of hyperbolic type established in [7,23] gradually forms an important subject in random dynamical systems [1,17,20,24], we refer to [25] for a detailed survey on this topic.

Coupled map lattices form a special class of infinite-dimensional dynamical systems. They were introduced by K. Kaneko [13] in 1983 as simple models with essential features of spatio-temporal chaos. These systems are built as weak interactions of identical local finite-dimensional subsystems at lattice points. Such systems are proven to be useful in studying qualitative properties of spatially extended dynamical systems. The first attempt to analyze coupled map lattices with multidimensional local subsystems of hyperbolic type was made by Pesin and Sinai in [28]. They constructed conditional distributions for the SRB-measure on unstable local manifolds assuming that the local subsystem possesses a hyperbolic attractor. In [8–12], the authors considered the case when a local subsystem possesses a hyperbolic set and obtained some partial results on the existence and uniqueness of Gibbs distributions. Coupled map lattices with unique Gibbs distribution can also be obtained under weaker conditions which is more natural from the physical point of view [16]. For resent progress of infinite-dimensional dynamical systems, we refer to [2,32,21,22].

Our principal aim in this article is to extend the equilibrium states theory for lattice models of hyperbolic type in [11] to the setting of random $\mathbb{Z}^2$ actions. More precisely, we aim to make a systematic study of the equilibrium states theory of lattice models under the conditions of random weak interaction and translation invariance over $\mathbb{Z}^2$ actions on Ω. To date, to the best of our knowledge, there has been little discussion of random lattice models with multidimensional local subsystems of hyperbolic type. We begin with a general result on structural stability of random coupled map lattice models of hyperbolic type under certain metrics. Our main obstacle is the multidimensional random thermodynamic formalism which is introduced in [18]. By establishing the relation of equilibrium states and Gibbs states, we prove the existence of equilibrium states for equi-Hölder continuous random functions on lattice models and touch only a few aspects of uniqueness of equilibrium state.

In the next section, we introduce the model. Our main results are given in Section 3, and Section 4 is devoted to the proofs.

2. Random lattice models

Let $f : M \to M$ be a $C^{1+\alpha}$ diffeomorphism ($\alpha > 0$) of a smooth compact finite dimensional Riemannian manifold without boundary, and $\Delta \subset M$ a compact locally maximal hyperbolic set for f. Those sets are also called Axiom A basic sets. Then, there is a continuous splitting of the tangent bundle $T_\Delta M = E^s \oplus E^u$, and constants $C > 0$ and $\lambda_0 \in (0, 1)$ such that for each $x \in \Delta$:

(A1) $d_x f E^s_x = E^s_{fx}$ and $d_x f E^u_x = E^u_{fx}$;

(A2) $||d_x f^n v|| \leqslant C\lambda_0^n ||v||$ for all $v \in E^s_x$ and $n \geqslant 0$;

(A3) $||d_x f^{-n} v|| \leqslant C\lambda_0^n ||v||$ for all $v \in E^u_x$ and $n \geqslant 0$.

Via a change of Riemannian metric we may-and-will always assume that $C = 1$.

For each point $x \in \Delta$ there exist local stable and unstable manifolds $W^s_{local}(x)$ and $W^u_{local}(x)$, with $T_x W^s_{local}(x) = E^s_x$ and $T_x W^u_{local}(x) = E^u_x$. Moreover, there exists $\delta > 0$ such that for all $x, y \in \Delta$ with $d(x, y) < \delta$, the set $W^s_{local}(x) \cap W^u_{local}(y)$ consists of a single point, which we denote by $[x, y]$, and the map $[\cdot, \cdot] : \{(x, y) \in \Delta \times \Delta : d(x, y) < \delta\} \to \Delta$ is continuous.

We may assume that $f|\Delta$ is topological mixing by replacing f with f^n, for some $n \in \mathbb{N}$.

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