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Solitary waves of a coupled KdV system with a weak rotation

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Abstract

Studied here is a coupled KdV system with a weak rotation. The existence of ground state solutions is proved, and the continuous dependence on the parameter and asymptotic behavior of ground state solutions are established. The orbital stability of ground states is also investigated.

MSC: 76B15; 86A05; 76U05; 76B25; 35B35

Keywords: KdV system with rotation; Solitary waves; Orbital stability; Variational problems

1. Introduction

In this paper we study a coupled KdV system with a weak rotation

$$\begin{cases} u_t - \alpha u_{xxx} - v^p v_x = 0, \\ (v_t - \kappa v_x - \beta v_{xxx} - (uv^p)_x)_x = \mu v \end{cases} \tag{1.1}$$

where α and β are the dispersion parameters, μ is the rotation parameter and $p = m/n$ with $m, n \in \mathbb{N}$, n is odd and m and n are relatively prime. System (1.1), when $p = 1$, was derived

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to describe weakly nonlinear oceanic internal waves when two distinct linear long wave modes have nearly coincident phase speeds [1]. This system was systematically derived for a density-stratified ocean. In the case $\mu = 0$, viz.

$$\begin{cases} u_t - \alpha u_{xxx} - v^p v_x = 0, \\ v_t - \kappa v_x - \beta v_{xxx} - (uv^p)_x = 0, \end{cases} \tag{1.2}$$

Gear and Grimshaw in [14] derived (1.2) to model interactions of long waves, for example in a stratified fluid. Some results on the global well-posedness of solution and the existence and stability of the solitary wave solutions of (1.2) were reported in [5,6].

Our aim here first is to consider the existence of solitary wave solution of (1.1). Using variational methods and the Pohozaev-type identities, we prove the existence and nonexistence of solitary waves for a range of the parameters of (1.1). We also show that our solitary waves are ground states, i.e. they have the least energy among all the non-trivial solutions. We also consider the effect of letting the rotation parameter μ or the dispersion parameter α approach zero. We also study the stability of solitary waves of (1.1) based on the following invariants:

$$\begin{aligned} E(u,v) &= \frac{1}{2}\int_{\mathbb{R}} \left(\alpha u_x^2 + \beta v_x^2 - \kappa v^2 + \mu(\partial_x^{-1} v)^2\right) \mathrm{d}x - \frac{1}{p+1}\int_{\mathbb{R}} uv^{p+1}\mathrm{d}x, \\ Q(u,v) &= \frac{1}{2}\int_{\mathbb{R}} (u^2+v^2)\mathrm{d}x, \end{aligned} \tag{1.3}$$

where $\partial_x^{-1} = \int_{-\infty}^{x}$. By a solitary wave of (1.1), we mean a solution of the form $u(x,t) = \varphi(x + c_1 t)$ and $v(x,t) = \psi(x + c_1 t)$. In order to study the existence and stability of solitary waves of (1.1), we first look for (φ, ψ) satisfying the following system of equations, which is obtained from (1.1) after replacing $x + c_1 t$ by x:

$$\begin{cases} c_1\varphi - \alpha\varphi'' - \frac{1}{p+1}\psi^{p+1} = 0, \\ (c_2\psi + \beta\psi'' + \varphi\psi^p)_{xx} + \mu\psi = 0, \end{cases} \tag{1.4}$$

where $c_2 + c_1 = \kappa$. In order to describe our results, we need to define a natural Lebesgue space.

Let H be the Banach space defined by $H = H^1(\mathbb{R}) \times \mathscr{X}$, equipped with the product norm $\|(f,g)\|_H^2 = \|f\|_{H^1(\mathbb{R})}^2 + \|g\|_{\mathscr{X}}^2$, for $(f,g) \in H$, where $\mathscr{X}$ is defined by

$$\mathscr{X} = \left\{ f \in H^1(\mathbb{R});\ (\xi^{-1}\widehat{f}(\xi))^\vee \in L^2(\mathbb{R}) \right\},$$

with the norm $\|f\|_{\mathscr{X}}^2 = \|f\|_{H^1(\mathbb{R})}^2 + \|\partial_x^{-1} f\|_{L^2(\mathbb{R})}^2$. It is worth noting from the Sobolev embedding for $u \in \mathscr{X}$ that $\partial_x^{-1} u \in L^q(\mathbb{R})$ for any $2 \le q \le \infty$.

Remark 1.1. It is well-known that the embedding $H^1(\mathbb{R}) \hookrightarrow L^q(\mathbb{R})$ is continuous for $2 \le q \le \infty$; and $H^1(\mathbb{R}) \hookrightarrow L^q_{\mathrm{loc}}(\mathbb{R})$ is compact for $2 \le q < \infty$. Similar results hold for the space $\mathscr{X}$ by using the density of $\mathscr{X}$ into $H^1(\mathbb{R})$ ([22, Lemma 3.3]). However, for $\Omega \subset \mathbb{R}$, this fact follows from the inequality

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