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On the heat kernel of a class of fourth order operators in two dimensions: Sharp Gaussian estimates and short time asymptotics

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Abstract

We consider a class of fourth order uniformly elliptic operators in planar Euclidean domains and study the associated heat kernel. For operators with L^∞ coefficients we obtain Gaussian estimates with best constants, while for operators with constant coefficients we obtain short time asymptotic estimates. The novelty of this work is that we do not assume that the associated symbol is strongly convex. The short time asymptotics reveal a behavior which is qualitatively different from that of the strongly convex case.

MSC: 35K25; 35E05; 35B40

Keywords: Higher order parabolic equations; Heat kernel estimates; Short time asymptotics

1. Introduction

Let Ω be a planar domain and let

$$Hu = \partial_{x_1}^2\big(\alpha(x)\partial_{x_1}^2 u\big) + 2\partial_{x_1}\partial_{x_2}\big(\beta(x)\partial_{x_1}\partial_{x_2} u\big) + \partial_{x_2}^2\big(\gamma(x)\partial_{x_2}^2 u\big)$$

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be a self-adjoint, fourth-order uniformly elliptic operator in divergence form on Ω with L^∞ coefficients satisfying Dirichlet boundary conditions on $\partial\Omega$. It has been proved by Davies [4] that H has a heat kernel $G(x,x',t)$ which satisfies the Gaussian-type estimate,

$$|G(x,x',t)| \le c_1 t^{-\frac{1}{2}} \exp\Big(-c_2 \frac{|x-x'|^{4/3}}{t^{1/3}} + c_3 t \Big), \tag{1}$$

for some positive constants c_1, c_2, c_3 and all $t > 0$ and $x, y \in \Omega$.

The problem of finding the sharp value of the exponential constant c_2 is related to replacing the Euclidean distance $|x-y|$ by an appropriate distance $d(x,y)$ that is suitably adapted to the operator H and, more precisely, to its symbol

$$A(x,\xi) = \alpha(x)\xi_1^4 + 2\beta(x)\xi_1^2\xi_2^2 + \gamma(x)\xi_2^4\,, \qquad x \in \Omega,\ \ \xi \in \mathbb{R}^2\,.$$

In the article [5], and for constant coefficient operators in $\mathbb{R}^n$ which satisfy suitable assumptions, the asymptotic formula

$$\begin{aligned} G(x,x',t) \sim h(x-x')^{-2/3} t^{-1/3} \exp\Big(-\frac{3\sqrt[3]{2}}{16} \frac{p_*(x-x')^{4/3}}{t^{1/3}} \Big) \\ \times \cos\Big(-\frac{3\sqrt{3}\sqrt[3]{2}}{16} \frac{p_*(x-x')^{4/3}}{t^{1/3}} - \frac{\pi}{3} \Big), \end{aligned} \tag{2}$$

was established as $t \to 0+$; here h is a positively homogeneous function of degree one and p_* is the Finsler metric defined by

$$p_*(\xi) = \max_{\eta \in \mathbb{R}^2 \setminus \{0\}} \frac{\eta \cdot \xi}{A(\eta)^{1/4}}. \tag{3}$$

An analogous asymptotic formula has been obtained in [7] in the more general case of operators with variable smooth coefficients; in this case the relevant distance is the (geodesic) Finsler distance $d_{p_*}(x,x')$ induced by the Finsler metric with length element $p_*(x,\xi)$, the latter being defined similarly to (3), with the additional dependence on x.

A sharp version of the Gaussian estimate (1) was established in [2] where it was proved that

$$|G(x,x',t)| \le c_\epsilon t^{-\frac{1}{2}} \exp\Big\{ -\Big(\frac{3\sqrt[3]{2}}{16} - D - \epsilon\Big) \frac{d_M(x,x')^{4/3}}{t^{1/3}} + c_{\epsilon,M} t \Big\}, \tag{4}$$

for arbitrary ϵ and M positive. Here $D \ge 0$ is a constant that is related to the regularity of the coefficients and $d_M(x,x')$, $M > 0$, is a family of Finsler-type distances on Ω which is monotone increasing and converges as $M \to +\infty$ to a limit Finsler-type distance $d(x,x')$ closely related to $d_{p_*}(x,x')$ but not equal to it; see also Subsection 3.1.

A fundamental assumption for both (2) and (4) is the *strong convexity* of the symbol $A(x,\xi)$ of the operator H. The notion of strong convexity was introduced in [5] where short time asymptotics were obtained not only for the operator described above but more generally for a constant coefficient operator of order $2m$ acting on functions on $\mathbb{R}^n$. In the context of the present article and for an operator with constant coefficients, strong convexity of the symbol $A(\xi)$ amounts to

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