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Homogenization of a nonlinear monotone problem with nonlinear Signorini boundary conditions in a domain with highly rough boundary

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Abstract

In this paper, we consider a domain $\Omega_\varepsilon \subset \mathbb{R}^N$, $N \geq 2$, with a very rough boundary depending on ε. For instance, if $N = 3$ Ω_ε has the form of a brush with an ε-periodic distribution of thin cylindrical teeth with fixed height and a small diameter of order ε. In Ω_ε we consider a nonlinear monotone problem with nonlinear Signorini boundary conditions, depending on ε, on the lateral boundary of the teeth. We study the asymptotic behavior of this problem, as ε vanishes, i.e. when the number of thin attached cylinders increases unboundedly, while their cross sections tend to zero. We identify the limit problem which is a nonstandard homogenized problem. Namely, in the region filled up by the thin cylinders the limit problem is given by a variational inequality coupled to an algebraic system.

MSC: 35B27; 35J60; 35R35

Keywords: Homogenization of rough boundaries; Nonlinear monotone problems; Nonlinear Signorini boundary conditions

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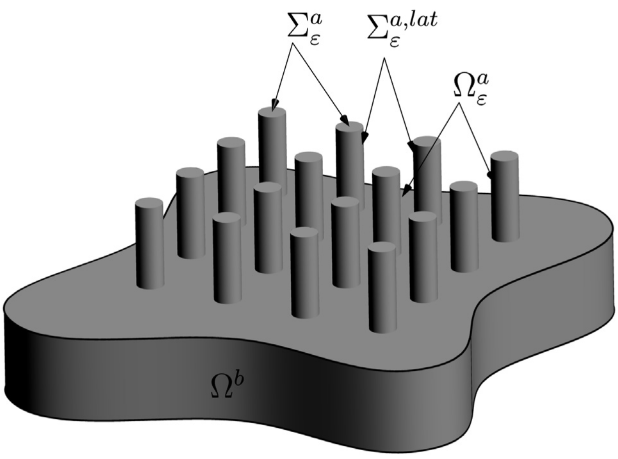

Fig. 1. Ω_ε.

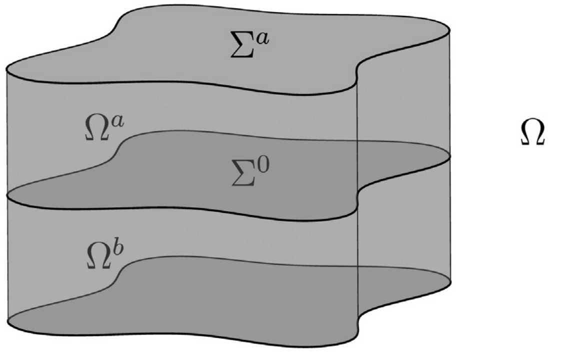

Fig. 2. Ω.

1. Introduction

This paper is devoted to studying the asymptotic behavior, as ε vanishes, of a nonlinear monotone problem with nonlinear Signorini boundary conditions, depending on ε, in a domain $\Omega_\varepsilon \subset \mathbb{R}^N$, $N \geq 2$, whose boundary contains a very rough part depending on ε.

The geometry of Ω_ε is rigorously introduced in Section 2. Roughly speaking, Ω_ε has the form of a brush in $3D$ (see Fig. 1) or the form of a comb in $2D$. It is composed of two parts: a fixed box Ω^b and a "forest" Ω_ε^a of cylinders with fixed height and small cross section of diameter of order ε, ε-periodically distributed in the first $N-1$ directions on the upper basis of Ω^b. The upper boundary and the lateral boundary of these cylinders are denoted by Σ_ε^a and $\Sigma_\varepsilon^{a,lat}$, respectively. Here as well as in the whole of the present paper, the superscripts a, b, and lat refer to "above", "below", and "lateral", respectively. Moreover, Ω^a denotes the "smallest" box containing Ω_ε^a for every ε, Σ^a and Σ^0 its upper basis and its lower basis, respectively, and $\Omega = \Omega^a \cup \Sigma^0 \cup \Omega^b$ (see Fig. 2).

Boundary-value problems in a domain with rough boundary arise in many fields of biology, physics and engineering sciences. For instance, for understanding the motion of ciliated mi-