Accepted Manuscript

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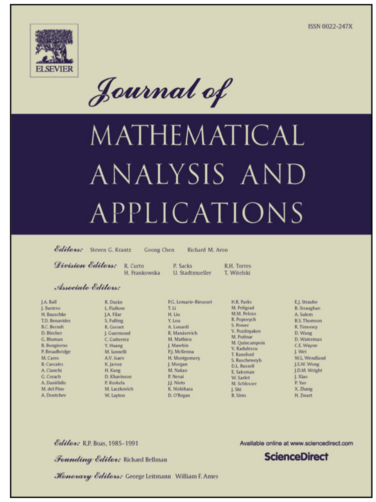

PII: S0022-247X(18)30626-7
DOI: https://doi.org/10.1016/j.jmaa.2018.07.044
Reference: YJMAA 22435

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 9 April 2018

Theory of Generalized Trigonometric Functions: from Laguerre to Airy Forms

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Abstract

We develop a new point of view to introduce families of functions, which can be identified as generalization of the ordinary trigonometric or hyperbolic functions. They are defined using a procedure based on umbral methods, inspired by the Bessel Calculus of Bochner, Cholewinsky and Haimo. We propose further extensions of the method and of the relevant concepts as well and obtain new families of integral transforms allowing the framing of the previous concepts within the context of generalized Borel transform.

Keywords: Trigonometric and Hyperbolic Functions, Laguerre Polynomials, Airy Forms, Umbral Calculus, Integral Transforms, Operational Methods.

1. Introduction

The cylindrical Bessel can be considered generalizations of the trigonometric functions, while the associated modified forms are an extension of the relevant hyperbolic counterparts [1].
Such an academic identification is non-particularly deep and might be useful for pedagogical reasons or as a guiding element to study their properties as e.g. those relevant to the asymptotic forms. We must however underline that Bessel and trigonometric/hyperbolic functions share some resemblances only, but they do not display any full correspondence.
The search for functions which are "true" generalizations of the trigonometric (t-) or hyperbolic (h-) forms is however recurrent in the mathematical literature. The attempts in this direction can be ascribed to different strategies, roughly speaking the geometrical [2] and the analytical [3] point of views.
The first is based on definitions extending to higher powers the Pythagorean identity of ordinary trigonometric functions, such a program identifies new trigonometries, with their own geometrical interpretation on elliptic curves and with different numbers playing the role of π [2].
The second invokes the analogy with series expansions, differential equations and the theory of special functions.

The generalized $t-h$ functions, defined within these two contexts, are different; in particular those belonging to the geometric strategy can be recognized as Elliptic functions, including Jacobi and Weierstrass forms. In this article we develop a systematic procedure within the framework of the analytical point of view.

We look for "true" generalizations, in the sense that the functions we define allow a one to one mapping onto the properties of the elementary $t-h$ functions, like addition or duplication theorems. To this aim we exploit the insightful point of view offered by the recent understanding of Bessel functions as umbral manifestation of Gauss or of exponential functions [4]. These conceptual tools, as well as the ideas developed by Cholewinsky and Reneke in ref. [5], provide the elements underlying the formalism of this paper, aimed at exploring in depth the identification of trigonometric functions associated with Bessel functions, by getting the proper algebraic environment to establish the

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