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Strong averaging principle for two-time-scale SDEs with non-Lipschitz coefficients ☆

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A B S T R A C T

This paper deals with averaging principle for two-time-scale stochastic differential equations (SDEs) with non-Lipschitz coefficients, which extends the existing results: from Lipschitz to non-Lipschitz case. Under suitable conditions, the existence of an averaging equation eliminating the fast variable for coupled system is established, and as a result, the system can be reduced to a single SDEs with a modified coefficient which is also non-Lipschitz. Moreover, it is shown that the slow variable strongly converges to the solution of the corresponding averaging equation.

1. Introduction

Consider the following the two-time-scale SDEs which was first studied by Khasminskii in [4]:

$$\begin{cases} \dot{X}_t^\epsilon = a(X_t^\epsilon, Y_t^\epsilon) + b(X_t^\epsilon)\dot{B}_t, \\ \dot{Y}_t^\epsilon = \frac{1}{\epsilon} f(X_t^\epsilon, Y_t^\epsilon) + \frac{1}{\sqrt{\epsilon}} g(X_t^\epsilon, Y_t^\epsilon)\dot{W}_t, \end{cases} \tag{1.1}$$

for $t > 0$, with initial conditions $X_0^\epsilon = x_0$ and $Y_0^\epsilon = y_0$, where X_t^ϵ is an n-dimensional diffusion process and Y_t^ϵ is an m-dimensional diffusion process. The driving processes B_t and W_t are d, l-dimensional independent Wiener processes defined on the probability space $(\Omega, \mathscr{F}, \mathbb{P})$ respectively. ϵ is a small positive parameter describing the ratio of time scale between the process X^ϵ and Y^ϵ. With this time scale the variable X^ϵ is referred as the *slow* component and Y^ϵ as the *fast* component. For the drift coefficients $a(u, v) : \mathbb{R}^n \times \mathbb{R}^m \to$

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$\mathbb{R}^n$, $f(u,v): \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^m$ and the diffusion coefficients $b(u): \mathbb{R}^n \to \mathbb{R}^n \times \mathbb{R}^d$, $g(u,v): \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^m \times \mathbb{R}^l$. Here the stochastic integrals are the usual Itô's integral.

The theory of stochastic averaging principle provides an effective approach for the qualitative analysis for complex systems with multiscales. In recently, E, Liu and Vanden-Eijnden [1] derived estimates for the rate of strong convergence to the solution of a couple fast variables ordinary differential system. After that, Liu proposed a generalization of the averaging principle for the case where both the slow and fast component were described by an Itô diffusion process (cf. [5]). In 2007, Givon [2] established a strong averaging principle for a system in which slow and fast dynamics were driven by Wiener process and Poisson process under suitable assumptions. Furthermore, Liu [6] extended the result of [2] to a couple fast variable in slow component case. In particular, Wainrib [10] developed an averaging principle for systems of slow-fast SDEs, where the fast variable drift was periodically modulated on a fast time-scale. All above work mainly discussed the averaging principle of SDEs for the two-time-scale diffusion process under the Lipschitz even much stronger conditions. We observe that the Lipschitz conditions do not conclude non-Lipschitz conditions in general. But, the non-Lipschitz conditions may derive Lipschitz conditions. Generally speaking, this is an irreversible process. If the averaging principle of a two-time-scale SDEs under the non-Lipschitz conditions is proved, then we may obtain a much bigger degree of freedom for choosing drift and diffusion coefficients in application. So, stochastic averaging principle of a two-time-scale SDEs with non-Lipschitz conditions is desired by people. However, strong averaging principle for two-time-scale SDEs with non-Lipschitz conditions has not been considered so far.

In the present paper, based on above discussions, we shall make the following assumptions:

(B1). The measurable functions a, b, c, f, g and h satisfy the non-Lipschitz conditions, i.e., there is a positive constant K_1 such that

$$\begin{aligned} &|a(u_1,v_1) - a(u_2,v_2)|^2 + \|b(u_1) - b(u_2)\|^2 \\ &\le K_1[|u_1-u_2|^2\kappa(|u_1-u_2|) + |v_1-v_2|^2\kappa(|v_1-v_2|)] \end{aligned} \tag{1.2}$$

and

$$\begin{aligned} &|f(u_1,v_1) - f(u_2,v_2)|^2 + \|g(u_1,v_1) - g(u_2,v_2)\|^2 \\ &\le K_1[|u_1-u_2|^2\kappa(|u_1-u_2|) + |v_1-v_2|^2\kappa(|v_1-v_2|)], \end{aligned} \tag{1.3}$$

where $\kappa(x) = (\log(1/x) \vee C)^{1/\beta}$ for some $\beta > 1$ and $C > 0$. Here and below we use $|\cdot|$ to denote Euclidean vector norms and $\|\cdot\|$ for Frobenius matrix norms.

(B2). There is a positive constant K_2 such that

$$|a(u,v)|^2 + \|b(u)\|^2 + |f(u,v)|^2 + \|g(u,v)\|^2 \le K_2(1+|u|^2+|v|^2), \tag{1.4}$$

for $(u,v) \in \mathbb{R}^n \times \mathbb{R}^m$.

(B3). There exists a constant $\alpha > 0$, which is independent of (u,v), such that

$$v^T g(u,v) g^T(u,v) v \ge \alpha |v|^2, \tag{1.5}$$

for all $(u,v) \in \mathbb{R}^n \times \mathbb{R}^m$.

(B4). There exist constants $q > 2$ and $\beta_1 > 0$, which are all independent of (u,v), such that

$$2v \cdot f(u,v) + \|g(u,v)\|^2 \le -\beta_1 |v|^q, \tag{1.6}$$

for all $u \in \mathbb{R}^n$ and $v \in \mathbb{R}^m$.