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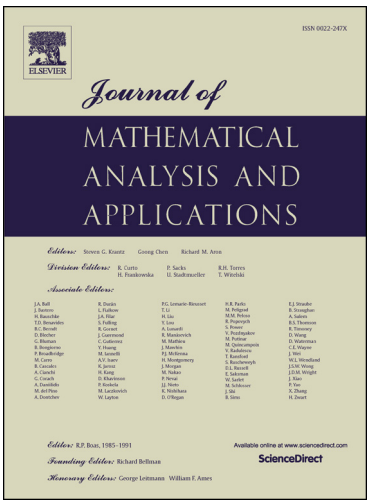

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Multiplicity results and qualitative properties for Neumann semilinear elliptic problems

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Abstract

In this paper we establish the existence of multiple ordered classical solutions for Neumann semilinear elliptic problems and provide qualitative information about them. No symmetry assumptions are required neither on the nonlinearity nor on the domain. For some results, the growth of the nonlinearity at infinity is arbitrary. We also consider the cases when the nonlinearity is either superlinear or aymptotically linear with strong resonances. Extensive use is made of the Mountain Pass Theorem and Leray-Schauder topological degree.

Keywords: Neumann boundary condition, Multiplicity of solutions, critical points of mountain pass type, local index, Leray-Schauder degree.

1. Introduction

In this paper we study existence of multiple nontrivial solutions and qualitative properties for the boundary value problem (BVP)

$$\begin{cases} -\Delta u = f(u) & \text{in} \quad \Omega, \\ \dfrac{\partial u}{\partial \nu} = 0 & \text{on} \quad \partial\Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^N$ $(N \geq 1)$ is a smooth and bounded domain with outer unit normal vector $\nu : \partial\Omega \to S^{N-1}$, the operator Δ is the Laplace operator and $f : \mathbb{R} \to \mathbb{R}$ is a continuously differentiable function.

Let

$$0 = \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_m \leq \cdots \longrightarrow +\infty$$

be the sequence of eigenvalues of $-\Delta$ in Ω with homogeneous Neumann boundary condition. For simplicity, from now on, we will denote by $\sigma(-\Delta_N)$ the set $\{\lambda_1, \lambda_2, ...\lambda_m, ...\}$. In order to describe some previous results, set $f'(+\infty) := \lim\limits_{t\to+\infty} \frac{f(t)}{t}$ and $f'(-\infty) := \lim\limits_{t\to-\infty} \frac{f(t)}{t}$, so that f is *asymptotically linear* if and only if $f'(\pm\infty) \in \mathbb{R}$.

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