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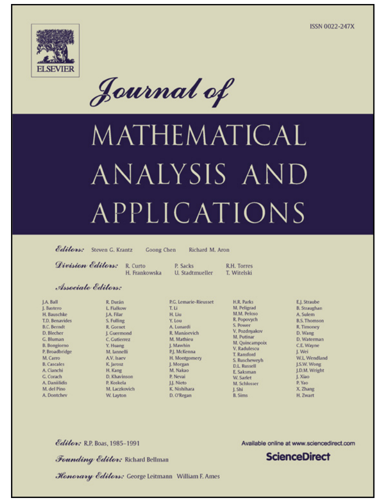

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PERIODICITY OF INHOMOGENEOUS TRAJECTORIES AND APPLICATIONS

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Abstract. We prove the existence and uniqueness of periodic solutions to inhomogeneous linear evolution equations and then give applications to Stokes equations in the space of bounded functions and to the inhomogeneous trajectories corresponding to semigroups which admit Gaussian estimates.

1. Introduction

The study of existence of T-periodic solutions to parabolic evolution equations of the form

$$u' - Au = f \tag{1.1}$$

for each given T-periodic forcing term f is an important research directions related to asymptotic behavior of solutions to differential equations. There some approaches used for that research, such as Massera principle [17, 24], Tikhonov's fixed point method [19] or the Lyapunov functionals [23] (which can be applied to some specific equations), and the most popular approaches for proving the existence of a periodic solution are the ultimate boundedness of solutions and the compactness of Poincaré map realized through some compact embeddings (see [5, 9, 19, 20, 21, 23] and the references therein). However, in some concrete applications, e.g., to partial differential equations in unbounded domains or to equations that have unbounded solutions, such compact embeddings are no longer valid, and the existence of bounded solutions is not easy to obtain since one has to carefully choose an appropriate initial vector (or conditions) to guarantee the boundedness of the solution emanating from that vector. One way to overcome such difficulties is to use the so-called Massera's type theorem, that is roughly speaking that if a differential equation has a bounded solution then it has a periodic one. Actually, we have invoked this Massera's methodology combining with interpolation spaces to prove the existence of periodic solutions to Navier-Stokes equations around a rotating obstacle in [18] and to general fluid flow problems in [6]. In those works we have used the interpolation functors in combination with ergodic method (see [18]) or with topological arguments (see [6]). Note that there is an approach described in [10] allowing $2\pi ki/T$ being in spectrum of A for some $k \in \mathbb{Z}$ under the requirement that such $2\pi ki/T$ are semisimple eingenvalues of A.

In the present paper, we propose another approach toward the existence and uniqueness of the periodic solution to the abstract evolution equation (1.1). Namely, we use the boundedness and smoothing properties of the corresponding semigroup to construct a Cauchy

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