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Van Duong Dinh

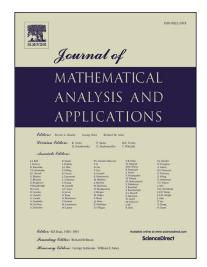
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## **ACCEPTED MANUSCRIPT**

# GLOBAL EXISTENCE AND BLOWUP FOR A CLASS OF THE FOCUSING NONLINEAR SCHRÖDINGER EQUATION WITH INVERSE-SQUARE POTENTIAL

#### VAN DUONG DINH

ABSTRACT. We consider a class of the focusing nonlinear Schrödinger equation with inverse-square potential

$$i\partial_t u + \Delta u - c|x|^{-2}u = -|u|^{\alpha}u, \quad u(0) = u_0 \in H^1, \quad (t, x) \in \mathbb{R} \times \mathbb{R}^d,$$

where  $d\geq 3$ ,  $\frac{4}{d}\leq \alpha\leq \frac{4}{d-2}$  and  $c\neq 0$  satisfies  $c>-\lambda(d):=-\left(\frac{d-2}{2}\right)^2$ . In the mass-critical case  $\alpha=\frac{4}{d}$ , we prove the global existence and blowup below ground states for the equation with  $d\geq 3$  and  $c>-\lambda(d)$ . In the mass and energy intercritical case  $\frac{4}{d}<\alpha<\frac{4}{d-2}$ , we prove the global existence and blowup below the ground state threshold for the equation. This extends similar results of [18] and [22] to any dimensions  $d\geq 3$  and a full range  $c>-\lambda(d)$ . We finally prove the blowup below ground states for the equation in the energy-critical case  $\alpha=\frac{4}{d-2}$  with  $d\geq 3$  and  $c>-\frac{d^2+4d}{(d+2)^2}\lambda(d)$ .

#### 1. Introduction

Consider the Cauchy problem for the focusing nonlinear Schrödinger equation with inverse-square potential

where 
$$u: \mathbb{R} \times \mathbb{R}^d \to \mathbb{C}, u_0: \mathbb{R}^d \to \mathbb{C}, d \geq 3, \alpha > 0$$
 and  $P_c = -\Delta + c|x|^{-2}$  with  $c \neq 0$  satisfies

where  $u: \mathbb{R} \times \mathbb{R}^d \to \mathbb{C}$ ,  $u_0: \mathbb{R}^d \to \mathbb{C}$ ,  $d \geq 3$ ,  $\alpha > 0$  and  $P_c = -\Delta + c|x|^{-2}$  with  $c \neq 0$  satisfies  $c > -\lambda(d) := -\left(\frac{d-2}{2}\right)^2$ . The case c = 0 is the well-known nonlinear Schrödinger equation which has been studied extensively over the last three decades. The nonlinear Schrödinger equation with inverse-square potential (NLS<sub>c</sub>) appears in a variety of physical settings and is of interest in quantum mechanics (see e.g. [14] and references therein). The study of the (NLS<sub>c</sub>) has attracted a lot of interest in the past several years (see e.g. [4, 18, 19, 20, 22, 26, 27, 29, 30, 34]).

The operator  $P_c$  is the self-adjoint extension of  $-\Delta + c|x|^{-2}$ . It is well-known that in the range  $-\lambda(d) < c < 1 - \lambda(d)$ , the extension is not unique (see e.g. [14]). In this case, we do make a choice among possible extensions, such as Friedrichs extension. The restriction on c comes from the sharp Hardy inequality, namely

$$\lambda(d) \int |x|^{-2} |u(x)|^2 dx \le \int |\nabla u(x)|^2 dx, \quad \forall u \in H^1, \tag{1.1}$$

which ensures that  $P_c$  is a positive operator.

Throughout this paper, we denote for  $\gamma \in \mathbb{R}$  and  $q \in [1, \infty]$  the usual homogeneous and inhomogeneous Sobolev spaces associated to the Laplacian  $-\Delta$  by  $\dot{W}^{\gamma,q}$  and  $W^{\gamma,q}$  respectively. We

1

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Key words and phrases. Nonlinear Schrödinger equation; Inverse-square potential; Global existence; Blowup; Virial identity; Gagliardo-Nirenberg inequality.

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