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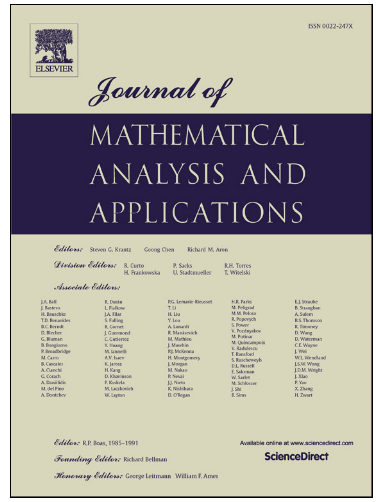

Global existence and blowup for a class of the focusing nonlinear Schrödinger equation with inverse-square potential

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GLOBAL EXISTENCE AND BLOWUP FOR A CLASS OF THE FOCUSING NONLINEAR SCHRÖDINGER EQUATION WITH INVERSE-SQUARE POTENTIAL

VAN DUONG DINH

Abstract. We consider a class of the focusing nonlinear Schrödinger equation with inverse-square potential

$$i\partial_t u + \Delta u - c|x|^{-2}u = -|u|^\alpha u, \quad u(0) = u_0 \in H^1, \quad (t,x) \in \mathbb{R}\times\mathbb{R}^d,$$

where $d \geq 3$, $\frac{4}{d} \leq \alpha \leq \frac{4}{d-2}$ and $c \neq 0$ satisfies $c > -\lambda(d) := -\left(\frac{d-2}{2}\right)^2$. In the mass-critical case $\alpha = \frac{4}{d}$, we prove the global existence and blowup below ground states for the equation with $d \geq 3$ and $c > -\lambda(d)$. In the mass and energy intercritical case $\frac{4}{d} < \alpha < \frac{4}{d-2}$, we prove the global existence and blowup below the ground state threshold for the equation. This extends similar results of [18] and [22] to any dimensions $d \geq 3$ and a full range $c > -\lambda(d)$. We finally prove the blowup below ground states for the equation in the energy-critical case $\alpha = \frac{4}{d-2}$ with $d \geq 3$ and $c > -\frac{d^2+4d}{(d+2)^2}\lambda(d)$.

1. Introduction

Consider the Cauchy problem for the focusing nonlinear Schrödinger equation with inverse-square potential

$$\begin{cases} i\partial_t u - P_c u &= -|u|^\alpha u, \quad (t,x)\in\mathbb{R}\times\mathbb{R}^d, \\ u(0) &= u_0, \end{cases} \tag{NLS$_c$}$$

where $u : \mathbb{R}\times\mathbb{R}^d \to \mathbb{C}$, $u_0 : \mathbb{R}^d \to \mathbb{C}$, $d \geq 3$, $\alpha > 0$ and $P_c = -\Delta + c|x|^{-2}$ with $c \neq 0$ satisfies $c > -\lambda(d) := -\left(\frac{d-2}{2}\right)^2$. The case $c = 0$ is the well-known nonlinear Schrödinger equation which has been studied extensively over the last three decades. The nonlinear Schrödinger equation with inverse-square potential (NLS$_c$) appears in a variety of physical settings and is of interest in quantum mechanics (see e.g. [14] and references therein). The study of the (NLS$_c$) has attracted a lot of interest in the past several years (see e.g. [4, 18, 19, 20, 22, 26, 27, 29, 30, 34]).

The operator P_c is the self-adjoint extension of $-\Delta + c|x|^{-2}$. It is well-known that in the range $-\lambda(d) < c < 1 - \lambda(d)$, the extension is not unique (see e.g. [14]). In this case, we do make a choice among possible extensions, such as Friedrichs extension. The restriction on c comes from the sharp Hardy inequality, namely

$$\lambda(d)\int |x|^{-2}|u(x)|^2dx \leq \int |\nabla u(x)|^2 dx, \quad \forall u \in H^1, \tag{1.1}$$

which ensures that P_c is a positive operator.

Throughout this paper, we denote for $\gamma\in\mathbb{R}$ and $q\in[1,\infty]$ the usual homogeneous and inhomogeneous Sobolev spaces associated to the Laplacian $-\Delta$ by $\dot{W}^{\gamma,q}$ and $W^{\gamma,q}$ respectively. We

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